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# 10.3 - The Newton-Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

① $f(x) = x^2 - 6x + 7$

a) $f(x) = 0 \Rightarrow f(1.5) = (1.5)^2 - 6(1.5) + 7 = 0.25 > 0$
$f(1.6) = (1.6)^2 - 6(1.6) + 7 = -0.04 < 0$

Sign change implies at least one root in interval

b) $f'(1.6) = f'(x) = 2x - 6 = 2(1.6) - 6 = -2.8$

c) $1.6 - \frac{f(1.6)}{f'(1.6)} = 1.6 - \frac{(-0.04)}{-2.8} = 1.585714$

d) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow a = 1 \quad b = -6 \quad c = 7 \Rightarrow x = 3 \pm \sqrt{2}$

e) $3 - \sqrt{2} = 1.585786$, so part c) answer correct to 3 dp

② $f(x) = x^4 - 3x^2 + x + 1$

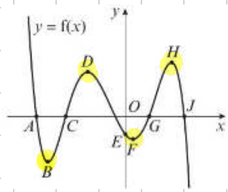
a) $f(1.2) = (1.2)^4 - 3(1.2)^2 + (1.2) + 1 = -0.464 < 0$
$f(1.3) = (1.3)^4 - 3(1.3)^2 + (1.3) + 1 = 0.0861 > 0$

Sign change implies at least one root in interval

b) $x_0 = 1.3 \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow f(x_0) = 0.0861$
$f'(x_0) = 4(1.3)^3 - 6(1.3) + 1 = 1.988$

$\Rightarrow 1.3 - \frac{0.0861}{1.988} = 1.257$

③




a) B, D, F, H are the points would not be valid as a first approximation as (B, D, F, H) are the turning point, $f'(x_i) = 0$.

④ $f(x) = 4\cos^2 x - e^{-x}$

a) $f(1.3) = 4\cos^2(1.3) - e^{-1.3} = 0.0136\ldots > 0$
$f(1.4) = 4\cos^2(1.4) - e^{-1.4} = -0.1310\ldots < 0$

Sign change implies at least one root in interval

b) $f'(x) = -8\sin x\cos x + e^{-x}$

c) $x_0 = 1.3 \Rightarrow x_{n+1} = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1.3 - \dfrac{f(1.3)}{f'(1.3)} = 1.308$

⑤ $f(x) = e^{-\frac{1}{4}x} + \dfrac{3}{4}x - \dfrac{1}{4}x^2$

a) $f(3.4) = e^{-0.25(3.4)} + 0.75(3.4) - 0.25(3.4)^2 = 0.0874\ldots > 0$

$f(3.5) = e^{-0.25(3.5)} + 0.75(3.5) - 0.25(3.5)^2 = -0.0206\ldots < 0$

Sign change implies at least one root in interval

b) $x_0 = 3.5 \Rightarrow x_{n+1} = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 3.5 - \dfrac{f(3.5)}{f'(3.5)} = 3.481$

c) $f(3.4805) = 0.0008\ldots > 0$ $\quad f(3.4815) = -0.0002\ldots < 0$

Sign change implies at least one root in interval, so $a = 3.481$ (3 dp)

⑥ $f(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{4}\ln x$

a) $f(1.2) = \dfrac{\cos^2(1.2)}{1.2} - \dfrac{1}{4}\ln(1.2) = 0.0638\ldots > 0$

$f(1.3) = \dfrac{\cos^2(1.3)}{1.3} - \dfrac{1}{4}\ln(1.3) = -0.0105\ldots < 0$

Sign change implies at least one root in interval.

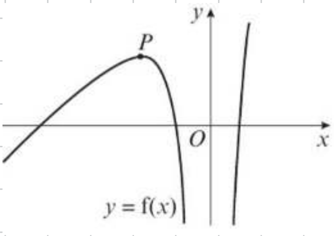
b $f'(x) = \left(\dfrac{\cos^2 x}{x}\right) - \dfrac{1}{4}\ln x$ $\Rightarrow u = \cos^2 x \quad v = x$

quotient rule $\quad u' = -2\cos x\sin x \quad v' = 1$

$$= \frac{-2x\sin x\cos x - \cos^2 x}{x^2} - \frac{1}{4x}$$

c) $x_0 = 1.3 \quad x_{n+1} = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1.3 - \dfrac{f(1.3)}{f'(1.3)} = 1.283$

⑦




$f(x) = x - \dfrac{4}{x^2} + 5$

$f'(x) = 0 \Rightarrow x^3 = -8 \quad \boxed{p = -2}$

$1 + \dfrac{8}{x^3} = 0 \quad x = -2$

$p = -2$; Gradient at point P is zero, so using Newton-Raphson formula would result in division by zero.

b) $f(0.8) = 0.8 - (4/0.8^2) + 5 = -0.45 < 0$
$f(0.9) = 0.9 - (4 - 0.9^2) + 5 = 0.961\ldots > 0$
Sign change implies at least one root in interval

c) $x_0 = -5$ $\quad x_{n+1} = x_0 - \dfrac{f(x_0)}{f'(x_0)} = -5 - \dfrac{f(5)}{f'(5)} = -4.829$