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Problem Solving : Set A   Chap 8

Bronze :

ai) $x = 10\sin t$, $y = 5\sin 2t$

$y = 5(2\sin t\cos t)$

$y = 10\sin t\cos t$

$y = x\cos t$

Hence, proved.

ii) Using : $\sin 2t = 2\sin t\cos t$ and

$\cos^2 t + \sin^2 t = 1$

$x = 10\sin t$

$\sin t = \frac{x}{10}$ — ①

$y = 5\sin 2t$

$y = 10\sin t\cos t$

$\cos t = \frac{y}{10\sin t}$

$\cos t = \frac{y}{x}$ — ②

Sub ① and ② in $\cos^2 t + \sin^2 t = 1$

$\left(\frac{y}{x}\right)^2 + \left(\frac{x}{10}\right)^2 = 1$

$\frac{y^2}{x^2} + \frac{x^2}{100} = 1$

$100y^2 + x^4 = 100x^2$

$100(y^2 - x^2) + x^2 = 0$

Hence, proved.

bi) Sub $t = \frac{\pi}{4}$ in x and y

$x = 10\sin\left(\frac{\pi}{4}\right)$, $y = 5\sin 2\left(\frac{\pi}{4}\right)$

$x = 5\sqrt{2}$ $y = 5$

So, coordinates of A are $(5\sqrt{2}, 5)$

Problem Solving: Set A Chap 8

Bronze:

bii] $y = x + K$

$5 = 5\sqrt{2} + K$

$K = 5 - 5\sqrt{2}$

Silver:

a] $y = 4\cos\left(t + \frac{\pi}{3}\right)$

$y = 4\cos t \cos\frac{\pi}{3} - 4\sin t \sin\frac{\pi}{3}$

$y = 2\cos t - \frac{4\sqrt{3}}{2}\sin t$

$y = 2\cos t - \frac{4\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}x\right)$

$y = 2\cos t - 3x$

$\cos t = \frac{y + 3x}{2}$, $\sin t = \frac{\sqrt{3}}{2}x$

Using $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{\sqrt{3}}{2}x\right)^2 + \left(\frac{y+3x}{2}\right)^2 = 1$

$(\sqrt{3}x)^2 + (y+3x)^2 = 2^2$

$(3x+y)^2 + 3x^2 - 4 = 0$

Hence, proved.

bi] When $t = \frac{\pi}{6}$

$x = \frac{2}{\sqrt{3}}\sin\frac{\pi}{6}$ $\quad x \Rightarrow \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

$y = 4\cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) \Rightarrow y = 0$

Problem Solving : Set A : Chap 8

Silver Cont.

b i] A $\left(\frac{\sqrt{3}}{3}, 0\right)$

When $t = \frac{\pi}{2}$

$x = \frac{2}{\sqrt{3}} \sin\frac{\pi}{2} \Longrightarrow x = \frac{2}{\sqrt{3}}$

$y = 4\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) \Longrightarrow y = -2\sqrt{3}$

B $\left(\frac{2\sqrt{3}}{3}, -2\sqrt{3}\right)$

$m = \dfrac{-2\sqrt{3} - 0}{\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{3}} \Longrightarrow \underline{\underline{-6}}$

ii] $y - y_1 = m(x - x_1)$

$y - 0 = -6\left(x - \frac{1}{\sqrt{3}}\right)$

$y = -6x + \frac{6}{\sqrt{3}}$

$y = -6x + 2\sqrt{3}$

$6x + y - 2\sqrt{3} = 0$

Gold :

a] $y = 2\sin t + \sin 2t$

$y = 2\sin t + 2\sin t\cos t$

$y = \sin t\,(2 + 2\cos t)$

$y = x\sin t$, $\cos^2 t = \frac{(x-2)^2}{\cancel{4}2}$

$y^2 = x^2\left(x - \frac{x^2}{4}\right)$

Problem Solving : Set A : Chap 8

Gold Cont.

a) $y^2 = x^3\left(1 - \frac{1}{4}x\right)$, $\sin^2 t = \frac{1}{1} - \frac{(x-2)^2}{4}$

When $t=0$, $x=4$ and $y=0$ $\quad = \frac{4-(x^2-4x+4)}{4}$

When $t = \frac{3\pi}{4}$, $\quad = \frac{-x^2+4x}{4}$

$x = 2-\sqrt{2}$ and $y = \sqrt{2}-1$ $\quad = \frac{x - x^2}{4}$

Therefore $a = 2-\sqrt{2}$ and $b = 4$

$$2-\sqrt{2} \le x \le 4$$

b) $A\ (2+\sqrt{2}\ ,\ 1+\sqrt{2})$

$$x + y = p$$

$$2 + \sqrt{2} + 1 + \sqrt{2} = p$$

$$3 + 2\sqrt{2} = p$$

c) at $t = \frac{3\pi}{4}$ $\Longrightarrow (-1+\sqrt{2}) + (2-\sqrt{2})$

$\Longrightarrow 1$

at $t = \frac{\pi}{4}$ $\Longrightarrow 3 + 2\sqrt{2}$

$$\{q : 1 \le q \le 3 + 2\sqrt{2}\}$$

Problem Solving: Set B: Chap 8

Bronze:

a) $x = 7.6t$, $y = -4.9t^2 + 16.3t$

When $y = 10$

$10 = -4.9t^2 + 16.3t$

$4.9t^2 \div 16.3t + 10$

So, $t = 2.515$ or $t = 0.8114$

$x = 19.1$ or $x = 6.17$

b) $x = 7.6t$ $\quad$ $y = -4.9t^2 + 16.3t$ —— ②

$t = \frac{x}{7.6}$ —— ①

Sub ① in ②

$$y = -4.9\left(\frac{x}{7.6}\right)^2 + 16.3\left(\frac{x}{7.6}\right)$$

$$y = \frac{-245}{2888}x^2 + \frac{163}{76}x$$

$$y = -x\left(\frac{245x}{2888} - \frac{163}{76}\right)$$

$x = 0$ or $\frac{245}{2888}x - \frac{163}{76} = 0$

$$x = \frac{6194}{245}$$

Max at $x = \frac{3097}{245}$

So, $y = -x\left(\frac{245}{2888}x - \frac{163}{76}\right)$

$$y = \frac{-3097}{245}\left(\frac{245}{2888}\left(\frac{3097}{245}\right) - \frac{1673}{76}\right)$$

$y = \underline{\underline{13.6\text{ m}}}$

Problem Solving : Set B : Chap 8

Silver :

a] When $t = 1$

$x = 16 \sin 4(1)$ $\quad$ $y = 32 \cos\left(8(1) - \frac{8\pi}{3}\right)$

$x = -12.11$ $\quad$ $y = 29.77$

distance $= \sqrt{(-12.11)^2 + (29.75)^2}$

$= 32.1 m$

b] When $t = 0$

$x = 16 \sin 4(0)$, $y = 32 \cos\left(8(0) - \frac{8\pi}{3}\right)$

$x = 0$ $\quad$ $y = -16$

$(0, -16)$

c] The period of $x(t)$ is $\frac{2\pi}{4} = \frac{\pi}{2}$

The period of $y(t)$ is $\frac{2\pi}{8} = \frac{\pi}{4}$

The least common multiple of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ is $\frac{\pi}{2}$.

Therefore, the time to complete one figure - eight is $\frac{\pi}{2}$ minutes

Gold :

a) $x = \frac{t^2 - 7t + 10}{t}$, $y = t \ln t$

When $x = 0$

$0 = t^2 - 7t + 10$

$0 = (t-5)(t-2)$

$t = 5$ or $t = 2$

Problem Solving : Set B : Chap 8

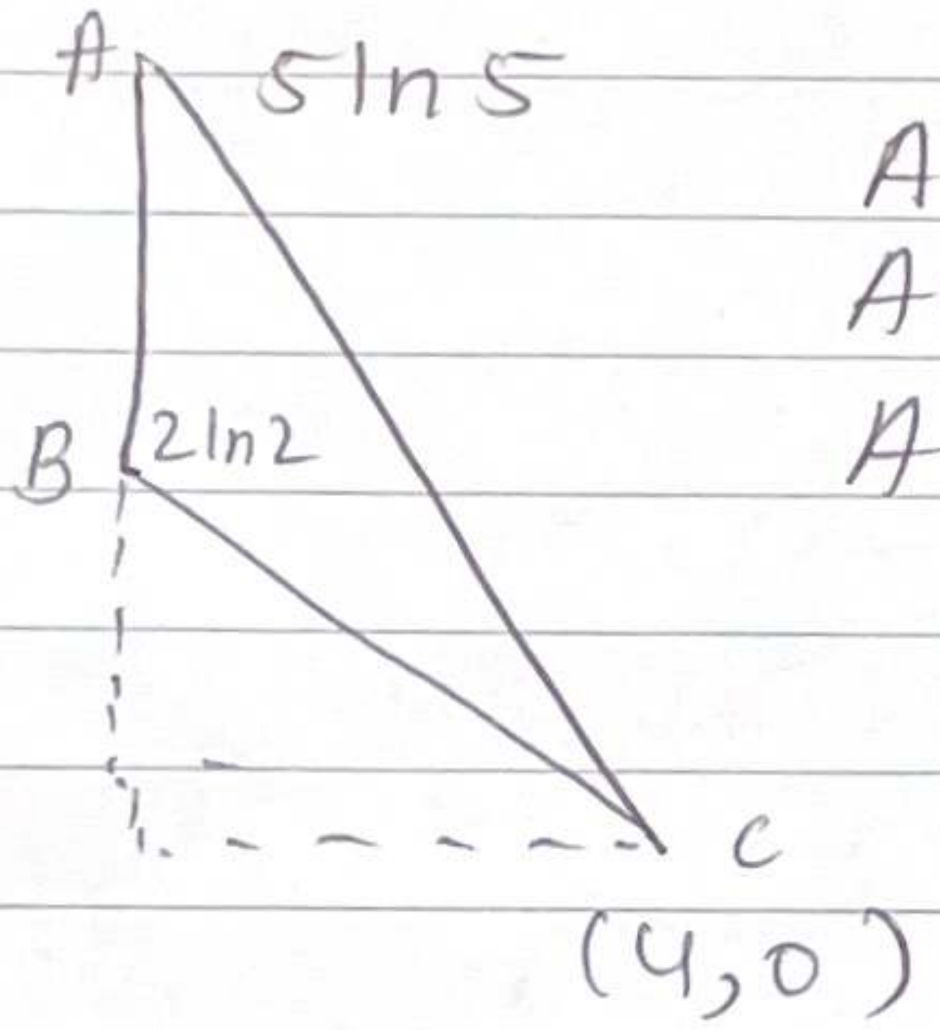
Gold Cont :

a]

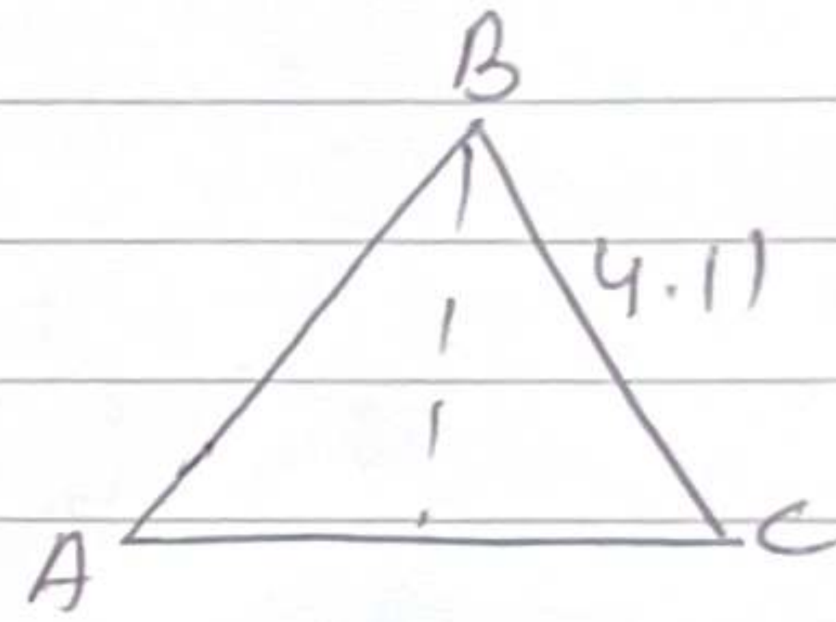
when $t = 5$, $y = 5\ln 5$
when $t = 2$, $y = 2\ln 2$

so $A(0, 5\ln 5)$
$B(0, 2\ln 2)$
$C(4, 0)$

when $y = 0$
$e^0 = t \implies t = 1$
when $t = 1 \implies x = (1)^2 - 7(1) + 10 \implies 4$



$AC^2 = 4^2 + (5\ln 5)^2$
$AC = \sqrt{4^2 + (5\ln 5)^2}$
$AC = 8.986$



$ar = \frac{1}{2} \times b \times h$

$ar = \frac{(5\ln 5 - 2\ln 2) \times 4}{2}$

$area = 10\ln 5 - 4\ln 2$
$= \ln\left(\frac{5^{10}}{2^4}\right)$
$= \ln\left(\frac{9765625}{16}\right)$

b]

$x = k$
$k = \frac{t^2 - 7t + 10}{t}$

$kt = t^2 - 7t + 10$
$t^2 - 7t \cancel{+} - kt + 10 = 0$
$t^2 + (-k - 7)t + 10 = 0$

Problem Solving : Set B : Chap 8

Gold Cont:

b) Using $b^2 - 4ac = 0$

$(-k-7)^2 - 4(1)(10) = 0$

$-k - 7 = \pm\sqrt{40}$

$k = -7 + \sqrt{40}$ or $-7 - \sqrt{40}$

$k = -0.675$ or -13.3

✓ ✗

(Closer to 0) (Too far from 0)