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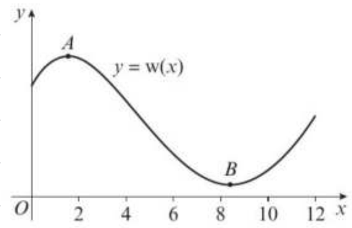
10·4 - Applications to modelling

① $w(x) = -\frac{1}{50}x^4 + \frac{7}{10}x^3 - 7x^2 + 17x + 40$



a) $w'(x) = -\frac{2}{25}x^3 + \frac{21}{10}x^2 - 14x + 17$

b) $w'(8.3) = -\frac{2}{25}(8.3)^3 + \frac{21}{10}(8.3)^2 - 14(8.3) + 17 = -0.27396 < 0$

$w'(8.4) = -\frac{2}{25}(8.4)^3 + 2.1(8.4)^2 - 14(8.4) + 17 = 0.15968 > 0$

There is a change of sign in the interval $8.3 < x < 8.4$, so there is at least one turning point in this interval.

c) turning point $\Rightarrow w'(x) = 0$

$\Rightarrow 0 = -\frac{2}{25}x^3 + \frac{21}{10}x^2 - 14x + 17$

$\Rightarrow \frac{21}{10}x^2 = \frac{2}{25}x^2 + 14x - 17$

$\Rightarrow x^2 = \frac{10}{21}\left(\frac{2}{25}x^2 + 14x - 17\right)$

$x = \pm\sqrt{\frac{10}{21}\left(\frac{2}{25}x^2 + 14x - 17\right)}$

$x_0 = 8.3 \quad x_{n+1} = \sqrt{\frac{10}{21}\left(\frac{2}{25}x_n^3 + 14x_n - 17\right)}$

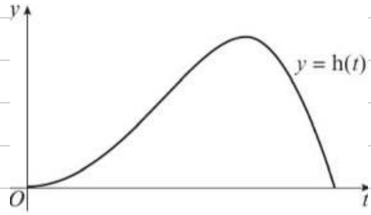
d) $x_1 = \sqrt{\frac{10}{21}\left(\frac{2}{25}x_0^3 + 14x_0 - 17\right)} = 8.308$

$x_2 = \sqrt{\frac{10}{21}\left(\frac{2}{25}x_1^3 + 14x_1 - 17\right)} = 8.315$

$x_3 = \sqrt{\frac{10}{21}\left(\frac{2}{25}x_2^3 + 14x_2 - 17\right)} = 8.321$

$x_4 = \sqrt{\frac{10}{21}\left(\frac{2}{25}x_3^3 + 14x_3 - 17\right)} = 8.326$

② $h(t) = 50\sin\left(\frac{t^2}{36}\right)$



a) $h(10) = 50\sin(10^2/36) = 17.7920... > 0$

$h(11) = 50\sin(11^2/36) = -10.8879... < 0$

Sign change implies at least one root in interval, therefore the boomerang lands between 10 and 11 seconds.

b) $h'(t) = 50\cos\left(\frac{t^2}{36}\right) \times \frac{d}{dt}\left(\frac{t^2}{36}\right) = 50\cos\left(\frac{t^2}{36}\right) \times \frac{2t}{36} = \frac{25t}{9}\cos\left(\frac{t^2}{36}\right)$

c) $x_0 = 1.5$ $\quad x_{n+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= 1.5 - \frac{h(1.5)}{h'(1.5)} = 1.5 - \frac{50\sin\left(\frac{1.5^2}{36}\right)}{\frac{25}{9}(1.5)\cos\left(\frac{1.5^2}{36}\right)}$

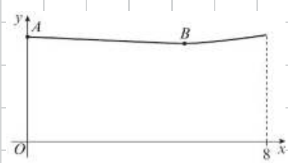
$= 10.64$ seconds

d) 7.520 $\quad$ LB 7.515 $\quad$ UP 7.525

$h(7.515) = 0.0425... > 0$ $\qquad h(7.525) = -0.4446... < 0$

Sign change implies at least one turning point in interval, so boomerang is a maximum height at 7.52 sec.

③ $p(x) = \frac{1}{3}\cos\left(\frac{x}{3}\right) + \frac{x^2}{100} + 1.29, \quad 0 \le x \le 8$



a) trading begins, $x = 0$, $p = £1.62$

b) $p'(x) = -\frac{1}{3}\sin\left(\frac{x}{3}\right) \times \frac{d}{dt}\left(\frac{x}{3}\right) + \frac{\cancel{2}x}{\cancel{100}\,50} \Rightarrow -\frac{1}{9}\sin\left(\frac{x}{3}\right) + \frac{x}{50}$

c) $p'(5) = -\frac{1}{9}\sin\left(\frac{5}{3}\right) + \frac{5}{50} = -0.0106... < 0$ $\quad p'(5.5) = -\frac{1}{9}\sin\left(\frac{5.5}{3}\right) + \frac{5.5}{50}$

$= 0.0026... > 0$

$\therefore$ Sign change implies at least one turning point in interval.

d) $-\frac{1}{9}\sin\left(\frac{x}{3}\right)+\frac{x}{50}=0 \Rightarrow \cancel{+}\frac{1}{9}\sin\left(\frac{x}{3}\right)=\cancel{+}\frac{x}{50} \Rightarrow x=\frac{50}{9}\sin\left(\frac{x}{3}\right)$

e) $x_{n+1}=\frac{50}{9}\sin\left(\frac{x_n}{3}\right)$; $x_0=5.3$

$x_1=\frac{50}{9}\sin\left(\frac{x_0}{3}\right)=5.449$

$x_2=\frac{50}{9}\sin\left(\frac{x_1}{3}\right)=5.389$

$x_3=\frac{50}{9}\sin\left(\frac{x_2}{3}\right)=5.415$

④ $v(t)=450e^{-0.5t}-40\cos t$

a) $v(5)=450e^{-0.5(5)}-40\cos(5)=25.591\ldots>0$
$v(6)=450e^{-0.5(6)}-40\cos(6)=-16.002\ldots<0$

Sign change implies at least one root in the interval

b) $v'(t)=-0.5(450e^{-0.5t})-(-40\sin t)=-225e^{-0.5t}+40\sin t$

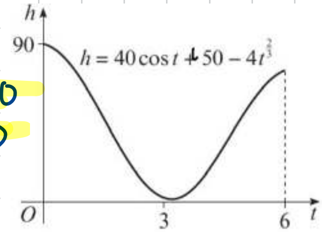
c) $x_0=5$ $\quad x_{n+1}=x_0-\frac{v(5)}{v'(5)} \Rightarrow 5-\frac{v(5)}{v'(5)}$

$\Rightarrow 5-\frac{(450e^{-0.5(5)}-40\cos(5))}{-225(e)^{-0.5(5)}+40\sin(5)}=5.4504$

$=5.45$ years

d) $v(6)=-16$, and the value cannot be negative, so this model is not suitable as the phone gets older than $\approx$ 5.5 years (approx)

⑤ $h = 40\cos t + 50 - 4t^{\frac{2}{3}}, \quad 0 \le t \le 6$



a) $h(2) = 40\cos(2) + 50 - 4(2)^{\frac{2}{3}} = 27.0045\ldots > 10$

$h(3) = 40\cos(3) + 50 - 4(3)^{2/3} = 2.0799\ldots < 10$

Therefore at some time between $t=2$ & $t=3$ the person is exactly 10m from the ground.

b) $t=3$, the person is 2.1 (approx) m from the ground, so the jump is not safe

c) $40\cos t + 50 - 4t^{\frac{2}{3}} = 10 \Rightarrow \cos t = \dfrac{4t^{\frac{2}{3}} + 10 - 50}{40}$

$\Rightarrow t = \arccos\left(\dfrac{4t^{\frac{2}{3}} - 40}{40}\right)$

d) $t_0 = 2.5$, $\quad t_{n+1} = \arccos\left(\dfrac{4t_n^{\frac{2}{3}} - 40}{40}\right)$

$t_1 = \arccos\left(\dfrac{4t_0^{2/3} - 40}{40}\right) = 2.52490$

$t_2 = \arccos\left(\dfrac{4t_1^{2/3} - 40}{40}\right) = 2.52280$

$t_3 = \arccos\left(\dfrac{4t_2^{2/3} - 40}{40}\right) = 2.52297$

$t_4 = \arccos\left(\dfrac{4t_3^{2/3} - 40}{40}\right) = 2.52296$