Author: Naga Karthik

This step-by-step solution guide has been created by **Naga Karthik** for educational purposes. While we have made every effort to ensure the accuracy of the information presented, it is possible that there may be errors or omissions. We encourage users to critically evaluate and verify the content. BF Maths and the author cannot be held responsible for any errors or inaccuracies in this guide.

If you find any mistakes or have any suggestions for improvements, please contact us at bfmathshello@gmail.com. Your feedback is invaluable in helping us maintain the quality and accuracy of our resources. Please specify *which exercise and which question* in the email.

Thank you for using BF Maths for your maths revision!

7.6 - Proving trigonometric identities

① a) $\dfrac{\sin 2x - \tan x}{\tan x} \equiv \cos 2x$

$\Rightarrow \dfrac{2\sin x \cos - \dfrac{\sin x}{\cos x}}{\dfrac{\sin x}{\cos x}} = \dfrac{\dfrac{2\sin x\cos^2 x - \sin x}{\cancel{\cos x}}}{\dfrac{\sin x}{\cancel{\cos x}}}$

$= \dfrac{\cancel{\sin x}\,(2\cos^2 x - 1)}{\cancel{\sin x}} = \cos 2x$

$\therefore$ LHS = RHS

$\boxed{\begin{aligned}&\cos 2x = \\ &2\cos^2 x - 1 = \\ &1 - 2\sin^2 x = \\ &\cos^2 x - \sin^2 x\end{aligned}}$

b) $\dfrac{\sin 6\theta}{1-\cos 6\theta} \equiv \cot 3\theta$

Let $3\theta = A$

$1 - \cos 2A = 2\sin^2 A$

$= \dfrac{\sin 2A}{1-\cos 2A} = \dfrac{\cancel{2\sin A}\cos A}{\cancel{2}\sin^{\cancel{2}} A} = \dfrac{\cos A}{\sin A} = \cot A$

$= \cot 3\theta$

$\therefore$ LHS = RHS

c) $\left(\dfrac{1}{4}\sec x \tan 2x \equiv \dfrac{1}{2}\sec 2x \sin x\right) \times 4$

$\Rightarrow \sec x\left(\dfrac{\sin 2x}{\cos 2x}\right) = 2\sec 2x \sin x \Rightarrow \dfrac{1}{\cos x}\left(\dfrac{\sin 2x}{\cos 2x}\right) = \dfrac{2}{\cos 2x}(\sin x)$

$\Rightarrow \dfrac{2\sin x\cancel{\cos x}}{\cancel{\cos x}\times\cos 2x} = \dfrac{2\sin x}{\cos 2x} \Rightarrow \dfrac{2\sin x}{\cos 2x} = \dfrac{2\sin x}{\cos 2x}$

$\therefore$ LHS = RHS

$\cos(A-B) \quad - \quad \cos(A+B)$

② a) $\cos(x - 30°) - \cos(x + 30°) \equiv \sin x$

$\Rightarrow \cos x\left(\dfrac{\sqrt{3}}{2}\right) + \sin x\left(\dfrac{1}{2}\right) - \left[\cos x\left[\dfrac{\sqrt{3}}{2}\right] - \sin x\left(\dfrac{1}{2}\right)\right]$

$$\cancel{\frac{\sqrt{3}}{2}\cos x} + \frac{1}{2}\sin x - \cancel{\frac{\sqrt{3}}{2}\cos x} + \frac{1}{2}\sin x = \sin x$$

$\therefore$ LHS = RHS

b) $$\frac{\sin(a+b)}{\sin(a-b)} \equiv \frac{\cot b + \cot a}{\cot b - \cot a}$$

$$= \frac{\sin a\cos b + \cos a \sin b}{\sin a \cos b - \sin b \cos a}$$ $\Rightarrow$ divide by $\sin a \sin b$

$$= \frac{\frac{\cancel{\sin a}\cos b}{\cancel{\sin a}\sin b} + \frac{\cos a\,\cancel{\sin b}}{\sin a\,\cancel{\sin b}}}{\frac{\cancel{\sin a}\cos b}{\cancel{\sin a}\sin b} - \frac{\cancel{\sin b}\cos a}{\cancel{\sin b}\sin a}} = \frac{\cot b + \cot a}{\cot b - \cot a}$$

$\therefore$ LHS = RHS

c) $$\cot\left[\frac{\pi}{4} - \theta\right] = \frac{1}{\tan\left[\frac{\pi}{4} - \theta\right]} = \frac{1}{\frac{\tan\left(\frac{\pi}{4}\right) - \tan\theta}{1 + \tan\left(\frac{\pi}{4}\right)\tan\theta}}$$

$$\tan\left[\frac{\pi}{4} - \theta\right] = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\theta}{1 + \tan\left(\frac{\pi}{4}\right)\tan\theta} = \frac{1 - \tan\theta}{1 + \tan\theta}$$

$$\Rightarrow \frac{1 - \frac{\sin\theta}{\cos\theta}}{1 + \frac{\sin\theta}{\cos\theta}} = \frac{\frac{\cos\theta - \sin\theta}{\cancel{\cos\theta}}}{\frac{\cos\theta + \sin\theta}{\cancel{\cos\theta}}} = \frac{1}{\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}}$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

$\therefore$ LHS = RHS

③ a) $2\sin\left[3\theta + \frac{\pi}{3}\right] = \sin 3\theta + \sqrt{3}\cos 3\theta$

$\sin(A+B)$ $\quad = 2\left[\sin 3\theta \cos\left(\frac{\pi}{3}\right) + \cos 3\theta \sin\left(\frac{\pi}{3}\right)\right]$

$= \cancel{2}\left[\frac{1}{\cancel{2}}\sin 3\theta + \frac{\sqrt{3}}{\cancel{2}}\cos 3\theta\right] = \sin 3\theta + \sqrt{3}\cos 3\theta$

$\therefore LHS = RHS$

b) $\cos 3A \equiv 4\cos^3 A - 3\cos A$

$= \cos(2A + A) = \cos 2A\cos A - \sin 2A \sin A$

$= (2\cos^2 A - 1)(\cos A) - 2\sin A\cos A(\sin A)$

$= 2\cos^3 A - \cos A - 2\sin^2 A\cos A$ $\quad \sin^2 A = 1 - \cos^2 A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)(\cos A)$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$

$\therefore LHS = RHS$

c) $\cot 2x + \tan x \equiv \operatorname{cosec} 2x$

$= \frac{1}{\tan 2x} + \tan x = \frac{1}{\sin 2x} \Rightarrow \frac{1-\tan^2 x}{2\tan x} + \tan x$

$= \frac{1 - \tan^2 x + 2\tan^2 x}{2\tan x} = \frac{1+\tan^2 x}{2\tan x} = \frac{\sec^2 x}{2\tan x}$

$= \dfrac{\frac{1}{\cos^2 x}}{\frac{2\sin x}{\cos x}} = \dfrac{1}{\frac{\cancel{\cos^2 x}}{\cos x}} \times \frac{\cancel{\cos x}}{2\sin x} = \frac{1}{2\sin x\cos x}$

$\Rightarrow \frac{1}{\sin 2x} = \operatorname{cosec} 2x$

$\therefore LHS = RHS$

④ $\sqrt{3}\cos x - \sin x = 2\cos(x+30^\circ)$ $(\cos(A+B))$

$= \cancel{2}\left[\cos x\left(\frac{\sqrt{3}}{\cancel{2}}\right) - \sin x\left(\frac{1}{\cancel{2}}\right)\right] = \sqrt{3}\cos x - \sin x$

$\therefore LHS = RHS$

⑤ a) $\frac{\sin 2x}{1+\cos 2x} \equiv \tan x$ $\quad \frac{2\sin x\cos x}{1+1-2\sin^2 x} = \frac{\cancel{2}\sin x\cos x}{\cancel{2}(1-\sin^2 x)}$

$= \frac{\sin x\cancel{\cos x}}{\cos^{\cancel{2}} x} = \tan x$

b) $\frac{\sin\frac{2\pi}{3}}{1+\cos\frac{2\pi}{3}} = \tan\frac{2\pi}{3} = -\sqrt{3}$

⑥ $2\sin\left[2\theta - \frac{\pi}{4}\right] \equiv 2\sqrt{2}\sin\theta\cos\theta - \sqrt{2} + 2\sqrt{2}\sin^2\theta$

$2\left[\sin 2\theta\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos 2\theta\right] = \cancel{2}\left[\frac{\sqrt{2}}{\cancel{2}}\sin 2\theta - \frac{\sqrt{2}}{\cancel{2}}\cos 2\theta\right]$

$= \sqrt{2}(2\sin\theta\cos\theta) - \sqrt{2}(1-2\sin^2\theta)$

$= 2\sqrt{2}\sin\theta\cos\theta - \sqrt{2} + 2\sqrt{2}\sin^2\theta$

⑦ a) $\underbrace{\cos^4 x - \sin^4 x}_{\text{Difference of square}} \equiv \cos 2x$

$= (\underbrace{\cos^2 x - \sin^2 x}_{\cos 2x})(\underbrace{\cos^2 x + \sin^2 x}_{1})$

$= \cos 2x$

b) $\tan x = \frac{1}{\sqrt{3}}$ $\qquad x = \frac{\pi}{6}$ $\qquad \cos 2x = \cos \cancel{2}\left(\frac{\pi}{\cancel{6}}\right) = \cos \frac{\pi}{3}$

$= \frac{1}{2}$