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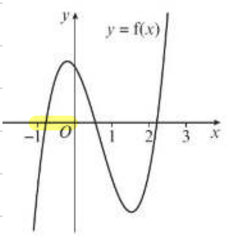
10.1 - Locating roots

① $f(x) = 2x^3 - 4x^2 - 2x + 2$



a) $x = -1$ and $x = 0$

$f(-1) = 2(-1)^3 - 4(-1)^2 - 2(-1) + 2 = -2 < 0$

$f(0) = 2(0)^3 - 4(0)^2 - 2(0) + 2 = 2 > 0$

Hence, there is a sign change in the interval $[-1, 0]$. there is a root.

b) i) $f(2.2) = 2(2.2)^3 - 4(2.2)^2 - 2(2.2) + 2 = -0.464$

ii) $f(2.3) = 2(2.3)^3 - 4(2.3)^2 - 2(2.3) + 2 = 0.574$

c) $f(2.2) = -0.464 < 0$ and $f(2.3) = 0.574 > 0$. Hence, there is a sign change in the interval $[2.2, 2.3]$. So, there is a root.

② a) $f(x) = 4 - \frac{1}{2}x^2 + 2x^3, \quad -1.2 < x < -1.1$

$\Rightarrow f(-1.2) = 4 - \frac{1}{2}(-1.2)^2 + 2(-1.2)^3 = -0.176 < 0$

$\Rightarrow f(-1.1) = 4 - \frac{1}{2}(-1.1)^2 + 2(-1.1)^3 = 0.733 > 0$

There is change of sign between $-1.2 < x < -1.1$. This implies at least one root.

b) $f(x) = x^2 \ln x - 2e^x + 4, \quad 0.6 < x < 0.7$

$\Rightarrow f(0.6) = (0.6)^2 \ln(0.6) - 2e^{0.6} + 4 = 0.1718\ldots > 0$

$\Rightarrow f(0.7) = (0.7)^2 \ln(0.7) - 2e^{0.7} + 4 = -0.2022\ldots < 0$

Sign change implies atleast one root in the interval $[0.6, 0.7]$

c) $f(x) = 3x^2 + \dfrac{5}{x^3}$, $-1.11 < x < -1.10$

$f(-1.11) = 3(-1.11)^2 + \dfrac{5}{(-1.11)^3} = 0.0403\ldots > 0$

$f(-1.10) = 3(-1.10)^2 + \dfrac{5}{(-1.10)^3} = -0.1265 < 0$

Sign change implies atleast one root in the interval $[-1.11, -1.10]$

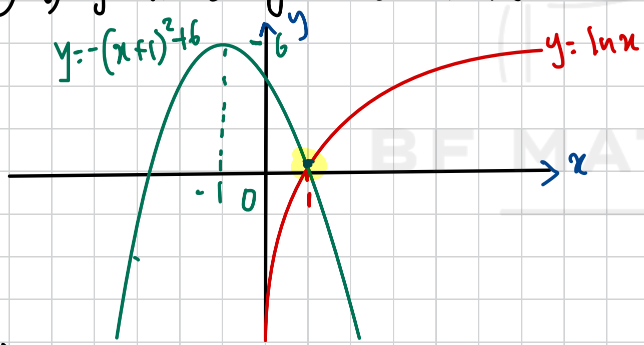
d) $f(x) = 9 + \dfrac{3}{4}x - 7\sqrt{x}$, $2.37 < x < 2.38$

$f(2.37) = 9 + \dfrac{3}{4}(2.37) - 7\sqrt{2.37} = 0.0011\ldots > 0$

$f(2.38) = 9 + \dfrac{3}{4}(2.38) - 7\sqrt{2.38} = -0.0140\ldots < 0$

Sign change implies atleast one root in the interval $[2.37, 2.38]$

③ a) $y = \ln x$ $\quad y = -(x+1)^2 + 6$



b) $\ln x = -(x+1)^2 + 6$

$\Rightarrow \ln x + (x+1)^2 - 6 = 0$

The graph of $y = \ln x$ and $y = -(x+1)^2 + 6$ only intersects once, so $f(x)$ has only one root.

c) $x = 1.3, 1.4$

$f(1.3) = \ln(1.3) + (1.3+1)^2 - 6 = -0.4476\ldots < 0$

$f(1.4) = \ln(1.4) + (2.4)^2 - 6 = 0.0964 \ldots > 0$

Sign change implies atleast one root in the interval $[1.3, 1.4]$

④ $f(x) = \cos^2 x - e^{-2x}$

a) $f'(x) = -2(\cos x)(\sin x) + 2e^{-2x}$

b) $f'(x) = -2\cos x \sin x + 2e^{-2x}$

$f'(3.1) = -2\cos(3.1)\sin(3.1) + 2e^{-2(3.1)} = 0.0871\ldots > 0$

$f'(3.2) = -2\cos(3.2)\sin(3.2) + 2e^{-2(3.2)} = -0.1132\ldots < 0$

As $f'(x)$ changes sign in the interval $[3.1, 3.2]$ there must be a stationary point in this interval.

c) $f'(3.1425) = 0.0019\ldots > 0$
$f'(3.1435) = -0.00009 \ldots < 0$

Sign change implies atleast one root in the interval $[3.1425, 3.1435)$, so $a = 3.143$

⑤ $g(x) = \frac{1}{2}x^3 - 2\cos x$

a) $g(1.1) = \frac{1}{2}(1.1)^3 - 2\cos(1.1) = -0.2416\ldots < 0$

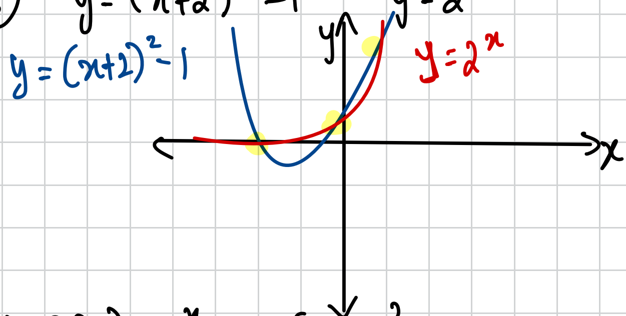
$g(1.2) = \frac{1}{2}(1.2)^3 - 2\cos(1.2) = 0.1392\ldots > 0$

Sign change implies atleast one root in the interval $[1.1, 1.2]$

b) $g(1.1645) = -0.0008\ldots < 0$
$g(1.1655) = 0.0030\ldots > 0$

There is a sign change $[1.1645, 1.1655]$ so, $a = 1.165$ (3dp)

⑥ $y = (x+2)^2 - 1 \qquad y = 2^x$



b) If $(x+2)^2 - 1 = 2^x$

As $y = (x+2)^2 - 1$ and $y = 2^x$ intersect thrice, so there are three roots to $f(x)$

c) $f(x) = 2^x + 1 - (x-2)^2$

$f(-0.8) = 2^{-0.8} + 1\ (-0.8-2)^2 = +0.1343\ldots > 0$
$f(0.7) = 2^{0.7} + 1\ (-0.7-2)^2 = -0.0744\ldots < 0$

} Sign change

⑦ $f(x) = \frac{2}{6-3x} - 4$

a) $f(1.9) = \frac{2}{6-3(1.9)} - 4 = \frac{8}{3} > 0$ | $f(2.1) = \frac{2}{6-3(2.1)} = -\frac{20}{3} < 0$

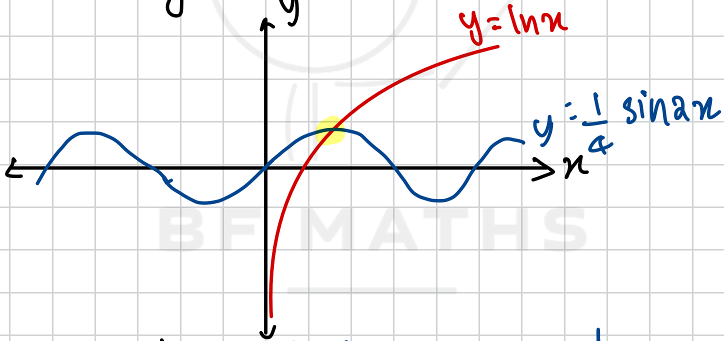
b) $f(2)$ = undefined (∞), as $= \frac{2}{6-3(2)} - 4 \Rightarrow \frac{2}{0} - 4$

c) $f(x) = 0 \Rightarrow \frac{2}{6-3x} - 4 = 0 \Rightarrow \frac{2}{6-3x} = 4$

$\Rightarrow 2 = 4(6-3x) \Rightarrow 2 = 24 - 12x \Rightarrow \cancel{-}22 = \cancel{-}12x$

$x = \frac{11}{6}$

⑧ a) $y = \frac{1}{4}\sin 2x \qquad y = \ln x$



b) $h(x) = \sin 2x - 4\ln x \Rightarrow$ As $y = \ln x$ and $y = \frac{1}{4}\sin 2x$ graph intersect only once. So, $h(x)$ has only one roots.

c) $h(1.18) = \sin 2(1.18) - 4\ln(1.18) = 0.0423\ldots > 0$

$h(1.19) = \sin 2(1.19) - 4\ln(1.19) = -0.0057\ldots < 0$

Sign change implies atleast one root in interval $[1.18, 1.19]$