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9.6 Differentiating trigonometric functions

1a] $y = \tan x$

$\frac{dy}{dx} = 4\sec^2 4x$

b] $y = 7\cot 2x$

$\frac{dy}{dx} = -14\operatorname{cosec}^2 2x$

c] $y = \cot^5 x \implies (\cot x)^5$

$\frac{dy}{dx} = 5(\cot x)^4(-\operatorname{cosec}^2 x)$

$= -5\cot^4 x \operatorname{cosec}^2 x$

d] $y = -3\tan^2 x \implies -3(\tan x)^2$

$\frac{dy}{dx} = -6(\tan x)\sec^2 x$

$= -6\tan x \sec^2 x$

2a] $y = \operatorname{cosec} 6x$

$\frac{dy}{dx} = -6\operatorname{cosec} 6x \cot 6x$

b] $y = -4\sec 5x$

$\frac{dy}{dx} = -4(5\sec 5x \tan 5x)$

$= -20\sec 5x \tan 5x$

c] $y = (\operatorname{cosec} x)^{4/3}$

$\frac{dy}{dx} = \frac{4}{3}(\operatorname{cosec} x)^{1/3}(-\operatorname{cosec} x \cot x)$

$= -\frac{4}{3}(\operatorname{cosec} x)^{4/3}\cot x$

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2d] $y = -2\sec^3 4x \implies -2(\sec 4x)^3$

$\frac{dy}{dx} = -6(\sec 4x)^2(4\sec 4x \tan 4x)$

$= -24\sec^3 4x \tan 4x$

3a] $x^2 \tan 3x$

$u = x^2$ $\quad$ $v = \tan 3x$

$\frac{du}{dx} = 2x$ $\quad$ $\frac{dv}{dx} = 3\sec^2 3x$

$\frac{dy}{dx} = 2x\tan 3x + 3x^2\sec^2 3x$

b] $\frac{e^{\sec x}}{x^2+1}$

$u = e^{\sec x}$ $\quad$ $v = x^2+1$

$\frac{du}{dx} = \sec x \tan x\, e^{\sec x}$ $\quad$ $\frac{dv}{dx} = 2x$

$\frac{dy}{dx} = \frac{(x^2+1)(\sec x \tan x\, e^{\sec x}) - 2x(e^{\sec x})}{(x^2+1)^2}$

$= \frac{(x^2+1)\sec x \tan x\, e^{\sec x} - 2xe^{\sec x}}{(x^2+1)^2}$

c] $\ln(\operatorname{cosec}^2 x)$

let $y = \ln(u)$, $u = \operatorname{cosec}^2 x$

$= (\operatorname{cosec} x)^2$

$\frac{du}{dx} = 2(\operatorname{cosec} x)(-\operatorname{cosec} x \cot x)$

$= -2\operatorname{cosec}^2 x \cot x$

So $\frac{dy}{dx} = \frac{1}{\cancel{\operatorname{cosec}^2 x}} \times (-2\cancel{\operatorname{cosec}^2 x}\cot x)$

$= -2\cot x$

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3d] $e^{-x} \cot 4x$

$u = e^{-x}$ $\qquad$ $v = \cot 4x$

$\frac{du}{dx} = -e^{-x}$ $\qquad$ $\frac{dv}{dx} = -4 \operatorname{cosec}^2 4x$

$\frac{dy}{dx} = -4e^{-x} \operatorname{cosec}^2 4x - e^{-x} \cot 4x$

$= -e^{-x} \cot 4x - 4e^{-x} \operatorname{cosec}^2 4x$

e] $\frac{\sec^2 x}{\ln x}$

$u = \sec^2 x$ $\qquad$ $v = \ln x$

$= (\sec x)^2$ $\qquad$ $\frac{dv}{dx} = \frac{1}{x}$

$\frac{du}{dx} = 2(\sec x)(\sec x \tan x)$

$= 2\sec^2 x \tan x$

$$\frac{dy}{dx} = \frac{\ln x (2\sec^2 x \tan x) - \frac{\sec^2 x}{x}}{(\ln x)^2}$$

$$= \frac{\frac{2x\sec^2 x \tan x (\ln x) - \sec^2 x}{x}}{(\ln x)^2}$$

$$= \frac{2x\sec^2 x \tan x (\ln x) - \sec^2 x}{x(\ln x)^2}$$

$$= \frac{\sec^2 x (2x \tan x \ln x - 1)}{x(\ln x)^2}$$

$$= \frac{\sec^2 x (2x \ln x \tan x - 1)}{x(\ln x)^2}$$

f] $\frac{(4x-1)^3}{\cot^2 x} \Longrightarrow \frac{(4x-1)^3}{(\cot x)^2}$

$u = (4x-1)^3$ $\qquad$ $v = (\cot x)^2$

$\frac{du}{dx} = 3(4x-1)^2(4)$ $\qquad$ $\frac{dv}{dx} = 2(\cot x)(-\operatorname{cosec}^2 x)$

$= 12(4x-1)^2$ $\qquad$ $= -2 \cot x \operatorname{cosec}^2 x$

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3b] Cont.

$$\frac{dy}{dx} = \frac{12(\cot x)^2(4x-1)^2 - (-2\cot x \operatorname{cosec}^2 x)(4x-1)^3}{((\cot x)^2)^2}$$

$$= \frac{12(\cot^2 x)(4x-1)^2 + 2\cot x \operatorname{cosec}^2 x\,(4x-1)^3}{\cot^4 x}$$

$$= \frac{12(4x-1)^2\cot^2 x + 2(4x-1)^3\cot x \operatorname{cosec}^2 x}{\cot^4 x}$$

4a] $x = \sin 2y$

$$\frac{dx}{dy} = 2\cos 2y$$

b] $$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$= \frac{1}{2\cos 2y}$$

Using identity $\sin^2 x + \cos^2 x = 1$ —— ①

Sub $x = 2y$ in equation ①

$$\sin^2 2y + \cos^2 2y = 1$$

$$\cos^2 2y = 1 - \sin^2 2y$$

$$\cos 2y = \sqrt{1 - \sin^2 2y}$$

Since $x = \sin 2y$

So $x^2 = \sin^2 2y$

So $\cos 2y = \sqrt{1 - x^2}$

So $$\frac{dy}{dx} = \frac{1}{2\cos 2y}$$

$$= \frac{1}{2\sqrt{1-x^2}}$$

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5] $y = \operatorname{cosec} x$

$y = \dfrac{1}{\sin x}$

$u = 1$ $\qquad$ $v = \sin x$

$\dfrac{du}{dx} = 0$ $\qquad$ $\dfrac{dv}{dx} = \cos x$

$$\frac{dy}{dx} = \frac{\sin x(0) - \cos x}{\sin^2 x}$$

$$= -\frac{\cos x}{\sin^2 x}$$

$$= -\frac{\cos x}{\sin x} \times \frac{1}{\sin x}$$

$$= -\cot x \operatorname{cosec} x$$

$$= -\operatorname{cosec} x \cot x$$

6] $f(x) = \ln(\cot 2x)$

a] $f'(x) = \dfrac{1}{\cot 2x} \times (-2\operatorname{cosec}^2 2x)$

$$= \frac{-2\operatorname{cosec}^2 2x}{\cot 2x}$$

$$= -2 \times \frac{1}{\sin^2 2x} \times \frac{\sin 2x}{\cos 2x}$$

$$= \frac{-2}{\sin 2x \cos 2x} \quad (\times 2)$$

$$= \frac{-4}{2\sin(2x)\cos(2x)}$$

Using identity

$\sin 2A = 2\sin A \cos A$

So $\sin 4x = 2\sin 2x \cos 2x$

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6a] Cont.

$$f'(x) = \frac{-4}{\sin 4x}$$

$$= -4\,\text{Cosec}(4x)$$

6b] $f'(x) = -4\,\text{Cosec}\,4x$

Sub $x = \frac{\pi}{8}$ in $f'(x)$

$$f'(x) = -4\,\text{Cosec}\,4\left(\frac{\pi}{8}\right)$$

$$= -4\,\text{Cosec}\,\frac{\pi}{2}$$

$$= \frac{-4}{\sin\frac{\pi}{2}}$$

$$= -4$$

Sub $x = \frac{\pi}{8}$ in $f(x)$

$$f(x) = \ln\left(\text{Cot}\,2\left(\frac{\pi}{8}\right)\right)$$

$$= \ln\left(\text{Cot}\,\frac{\pi}{4}\right)$$

$$= \ln(1)$$

$$= 0$$

So $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - 0 = \frac{-1}{-4}\left(x - \frac{\pi}{8}\right)$$

$$y = \frac{x}{4} - \frac{\pi}{32}$$

$$y = \frac{1}{4}x - \frac{\pi}{32}$$

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7] $f(x) = \cancel{\tan\frac{1}{2}x}\ e^{\tan\frac{1}{2}x}$

when $x = 0$

$f(x) = e^{\tan\frac{1}{2}(0)}$

$f(x) = 1$

So $A\ (0, 1)$

$f'(x) = \frac{1}{2}\sec^2\frac{1}{2}x\ e^{\tan\frac{1}{2}x}$

when $x = 0$

$f'(x) = \frac{1}{2}\sec^2\frac{1}{2}(0)\, e^{\tan\frac{1}{2}(0)}$

$= \frac{1}{2} \times 1$

$= \frac{1}{2}$

So $m = -2$

So, $y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x + 1$ ——— ①

So, $y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 1 = -2(x - 0)$

$y = -2x + 1$ ——— ②

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7] Cont.

Using equation ①

when $y = 0$

$$0 = \frac{1}{2}x + 1$$

$$x = -2$$

So $B(-2, 0)$

Using equation ②

Sub $y = 0$

$$0 = -2x + 1$$

$$x = \frac{1}{2}$$

So, $C\left(\frac{1}{2}, 0\right)$

So, area of the triangle

$$= \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times \frac{5}{2} \times 1$$

$$= \frac{5}{4}$$

So, area of triangle $= \frac{5}{4}$

8] $x = \sec 4y$

a] $\frac{dx}{dy} = 4\sec 4y \tan 4y$

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8b] $\frac{dy}{dx} = \frac{1}{4\sec 4y \tan 4y}$

Since $x = \sec 4y$

sub it into $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{4x\tan 4y}$

Using identity

$1 + \tan^2 A = \sec^2 A$

$1 + \tan^2 4y = \sec^2 4y$

$\tan^2 4y = \sec^2 4y - 1$

$\tan 4y = \sqrt{\sec^2 4y - 1}$

Since $x = \sec 4y$

So, $x^2 = \sec^2 4y$

$\tan 4y = \sqrt{x^2 - 1}$

So $\frac{dy}{dx} = \frac{1}{4x\sqrt{x^2-1}}$