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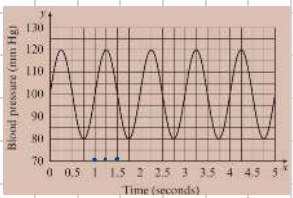
Ch:7 - Trigonometry and modelling

SET A

BRONZE:



$y = a + b\sin(cx)^\circ$

a) $100 = a + b\sin(c(0))^\circ$

$100 = a + b(0)$

$\boxed{a = 100}$

$a + b = 120$ ($\sin(cx^\circ) = 1$, max)

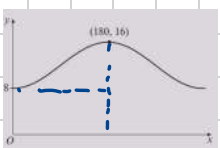
$100 + b = 120$

$\Rightarrow \boxed{b = 20}$

b) $T = 1$; The period T is the time it takes for the curve to complete one cycle.

$T = \frac{360^\circ}{c} \Rightarrow 1 = \frac{360^\circ}{c}$ $\boxed{c = 360^\circ}$ $\Rightarrow y = 100 + 20\sin(360^\circ x)$

SILVER:



$y = a + b\sin(x - c)^\circ$

a) $a = \frac{16+8}{2} = 12$ $b = \frac{16-8}{2} = 4$ $c = \frac{180^\circ}{2} = 90^\circ$

$\Rightarrow y = 12 + 4\sin(x - 90^\circ)$

b) Two transformation are a vertical shift of 12 units upward and a horizontal shift of 90 units to the right

c) $12 + 4\sin(x - 90^\circ) > 14 \Rightarrow \frac{2}{\cancel{4}}\sin(x - 90^\circ) > \frac{\cancel{2}}{2} \Rightarrow \sin(x - 90^\circ) > \frac{1}{2}$

$x - 90^\circ = 30^\circ, 150^\circ \Rightarrow x = 120, 240 \Rightarrow 121 - 2 = 119$ days

GOLD:

$h = a + b\cos(ct - d)^\circ$; Intial height: (0, 2) ; max: (22, 90) ; period = 4

$\Rightarrow h(0) = a + b\cos(-d) = 2$

$\Rightarrow h(90) = a + b\cos(c90^\circ - d) = 22$

$\cos c.90^\circ - d = 1 \Rightarrow a + b = 22$

$\boxed{b = 22 - a}$

$\Rightarrow a + 22 - a\cos(-d) = 2$

$22\cos -d + a\cos(-d) + a = 2$

$\cos -d = k$

$22k - a(1 - k) = 2$

$T = 12\text{min}\ (12 \times 60 = 720\ \text{sec}) = \frac{720}{4} = 180^\circ \Rightarrow C = \frac{360^\circ}{T} \Rightarrow C = \frac{360^\circ}{180^\circ} = 2$

$\Rightarrow \underset{(\text{max})}{C \times 90 - d = 0} \Rightarrow 2 \times 90^\circ = d \quad \boxed{d = 180^\circ}$

$\Rightarrow \cos(-180^\circ) = -1 \Rightarrow a - b = 2 \text{ -①} \Rightarrow a + b = 22 \text{ -②}$

$\Rightarrow b = 22 - 12 = 10$

$a + b = 22$
$a - b = 2$
$2a = 24$

$\boxed{a = 12}$

$\Rightarrow y = 12 + 10\cos(2t - 180^\circ)$

b) $T = \frac{720}{5} = 144 \qquad C = \frac{360^\circ}{144^\circ} = \frac{5}{2} \Rightarrow y = 12 + 10\cos\left(\frac{5}{2}t - 180^\circ\right)$

C) Revathi is correct. As she says that the length of time the car is more than 20m above the ground will be the same for both models.

SET B

BRONZE

a) $12\sin 2x - 5\cos 2x \Rightarrow R\sin(2x-a)$

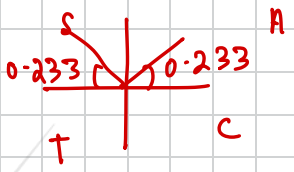
$\Rightarrow R = \sqrt{12^2+5^2} = 13$; $\tan a = \frac{5}{12}$ $a = 0.395$ $\Rightarrow$ $13\sin(2x-0.395)$

b) $12 - 3\operatorname{cosec} 2x = 5\cot 2x \Rightarrow 12 - \frac{3}{\sin 2x} = \frac{5\cos 2x}{\sin 2x} \Rightarrow \frac{12\sin 2x - 3}{\sin 2x} = \frac{5\cos 2x}{\sin 2x}$

$\Rightarrow 12\sin 2x - 5\cos 2x = 3$

c) $12\sin 2x - 5\cos 2x = 3 \Rightarrow 13\sin(2x-0.395) = 3 \Rightarrow \sin(2x-0.395) = \frac{3}{13}$

$0 \le 2x \le 4\pi$



$2x - 0.395 = 0.233, \pi - 0.233, 2\pi + 0.233, 3\pi - 0.233.$

$2x - 0.395 = 0.233, 2.91, 6.52, 9.19$

$x = 0.3, 1.7, 3.5, 4.8$ (1dp)

SILVER

a) $7\cos^2 x - 3\sin^2 x - 8\sin x\cos x$

$\Rightarrow \cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$; $\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$

$\Rightarrow 7\cos^2 x = 7\left(\frac{1+\cos 2x}{2}\right) = \frac{7+7\cos 2x}{2}$

$\Rightarrow -3\sin^2 x = -3\left(\frac{1-\cos 2x}{2}\right) = \frac{3-3\cos 2x}{2}$

$\Rightarrow 7\cos^2 x - 3\sin^2 x = \frac{7+7\cos 2x+3-3\cos 2x}{2} = \frac{4+10\cos 2x}{2} = 2+5\cos 2x$

$\Rightarrow -8\sin x\cos x \Rightarrow -4(2\sin x\cos x) \Rightarrow -4\sin 2x$

$\Rightarrow 5\cos 2x - 4\sin 2x + 2$

b) $7\left(\frac{1+\cos 2x}{2}\right) - 3\left(\frac{1-\cos 2x}{2}\right) - 4\sin 2x = 4 \Rightarrow 10\cos 2x - 8\sin 2x = 4$

$\frac{7+7\cos 2x - 3 + 3\cos 2x}{2} - 4\sin 2x = 4$

$\Rightarrow 4 + 10\cos 2x - 8\sin 2x - 8 = 0$

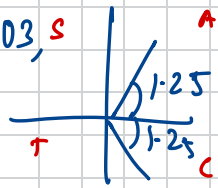
$10\cos 2x - 8\sin 2x = 4 \Rightarrow 10\cos 2x - 8\sin 2x = 4$

$R\cos(2x+a) = 4$

$\Rightarrow R = 2\sqrt{41} \Rightarrow a = 0.67 \Rightarrow 2\sqrt{41}\cos(2x+0.67) = 2$

$\Rightarrow \cos(2x+0.67) = \frac{2}{\sqrt{41}}$

$2x + 0.67 = 1.25, 5.03, 7.53, 11.8$



$\Rightarrow x = 0.29, 2.18, 3.43, 5.32$

c) i) $10\cos^2 x - 8\sin x\cos x = 10\left(\frac{1+\cos 2x}{2}\right) - 4(2\sin x\cos x)$

$\Rightarrow 5\cos 2x + 5 - 4\sin 2x \Rightarrow 5\cos 2x - 4\sin 2x + 5$

ii) Max value :→ $5\cos 2x - 4\sin 2x + 5$

$\sqrt{41}\cos(2x+0.67) + 5$

Max value: $\cos(2x+0.67) = 1 \Rightarrow \sqrt{41}(1) + 5 = 5 + \sqrt{41}$

GOLD:

a) $\frac{15}{2}\sin 4x - 4\cos 4x - 4\cos 2x - 4$

$\Rightarrow \frac{15}{2}\sin 4x = 4\cos 4x + 4\cos 2x + 4$

$\Rightarrow \frac{15}{2}\sin 4x = 4(\cos 4x + \cos 2x + 1)$

$\Rightarrow 15\sin 4x = 8(\cos 4x + \cos 2x + 1)$

$\Rightarrow 15(2\sin 2x\cos 2x) = 8(2\cos^2 2x - 1 + 1 + \cos 2x)$

$\Rightarrow 15\sin 2x\cos 2x - 8\cos^2 2x - 4\cos 2x = 0$

$\Rightarrow \cos 2x(15\sin 2x - 8\cos 2x - 4) = 0$

$a = 15 \quad b = 8 \quad c = 4$

b) $\cos 2x = 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

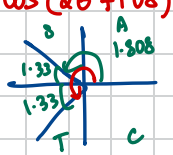
$15\sin 2x = 8\cos 2x + 4$ $\quad 0 \le 2x \le 2\pi$

$\Rightarrow 8\cos 2x - 15\sin 2x + 4$

$R\cos(2x+a) \Rightarrow R = 17 \quad a = 1.08 \Rightarrow 17\cos(2\theta + 1.08) = -4$

$2\theta + 1.08 = 1.808, 4.475$

$\Rightarrow \theta = 0.36, 1.70$



$x = 0.36, 0.79, 1.70, 2.36$

c) $\frac{1}{30\sin x\cos x - 16\cos^2 x} = \frac{1}{15(2\sin x\cos x) - 8(2\cos^2 x - 1)}$

$\Rightarrow \frac{1}{15\sin 2x - 8\cos 2x - 8} = \frac{1}{17\sin(2x+a) - 8} = \frac{1}{17(1) - 8} = \frac{1}{17-8} = \frac{1}{9}$

↳ $\sin(2x+a) = 1$, when x is max