Author: Blinzy Fernandes

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8.2 Using trigonometric identities

1a) $x = \sin t - 1$ $\qquad$ $y = \cos t + 3$

$x + 1 = \sin t$ $\qquad$ $y - 3 = \cos t$

Using $\sin^2 t + \cos^2 t = 1$

$(x+1)^2 + (y-3)^2 = 1$

b) $x = 2\sin t$ $\qquad$ $y = 3\cos t$

$\frac{x}{2} = \sin t$ $\qquad$ $\frac{y}{3} = \cos t$

Using $\sin^2 t + \cos^2 t = 1$

$= \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

c) $x = 2 + 2\cos t$ $\qquad$ $y = 5 + 2\sin t$

$\frac{x-2}{2} = \cos t$ $\qquad$ $\frac{y-5}{2} = \sin t$

Using $\sin^2 t + \cos^2 t = 1$

$\left(\frac{y-5}{2}\right)^2 + \left(\frac{x-2}{2}\right)^2 = 1$

$(y-5)^2 + (x-2)^2 = 4$

2a) $x = \sin t$ $\qquad$ $y = \cos 2t$

Using $\cos 2t = 1 - 2\sin^2 t$

$y = 1 - 2x^2$

b) $x = 4\sec t$, $\qquad$ $y = 2\tan t$

$\frac{x}{4} = \sec t$ $\qquad$ $\frac{y}{2} = \tan t$

Using $1 + \tan^2 t = \sec^2 t$

8.2 Using trigonometric identities

2b) Cont.

$$1 + \left(\frac{y}{2}\right)^2 = \left(\frac{x}{4}\right)^2$$

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

2c) $x = \sin t$ $\quad y = \sin^2 2t$

$\sqrt{y} = \sin 2t$

Using $\sin 2t = 2\sin t \cos t$

$\sqrt{y} = 2x \cos t$

Square on both sides

$(\sqrt{y})^2 = (2x\cos t)^2$

$y = 4x^2 \cos^2 t$

Using $\cos^2 t = 1 - \sin^2 t$

$y = 4x^2(1 - \sin^2 t)$

$y = 4x^2(1 - x^2)$

$y = 4x^2 - 4x^4$

3ai) $x = \sin t + 3$ $\quad y = \cos t - 4$

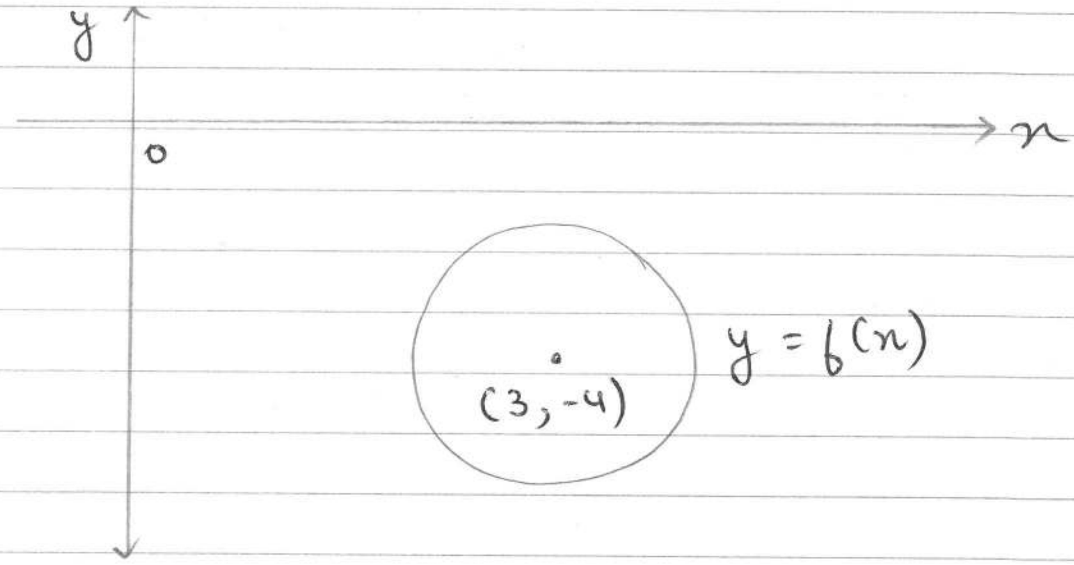
$x - 3 = \sin t$ $\quad y + 4 = \cos t$

Using $\sin^2 t + \cos^2 t = 1$

$(x-3)^2 + (y+4)^2 = 1$

8.2 Using trigonometric identities

3a ii]



b i]

$$x = \frac{2}{\tan t} \qquad y = 4\operatorname{cosec}^2 t - 8$$

$$\tan t = \frac{2}{x} \qquad \therefore\ y + 8 = 4\operatorname{cosec}^2 t$$

$$\frac{y+8}{4} = \operatorname{cosec}^2 t$$

Using $\operatorname{cosec}^2 t = 1 + \cot^2 t$

$$\operatorname{cosec}^2 t = 1 + \frac{1}{\tan^2 t}$$

$$\frac{y+8}{4} = 1 + \frac{1}{\left(\frac{2}{x}\right)^2}$$

$$\frac{y+8}{4} = 1 + \frac{1}{\frac{4}{x^2}}$$

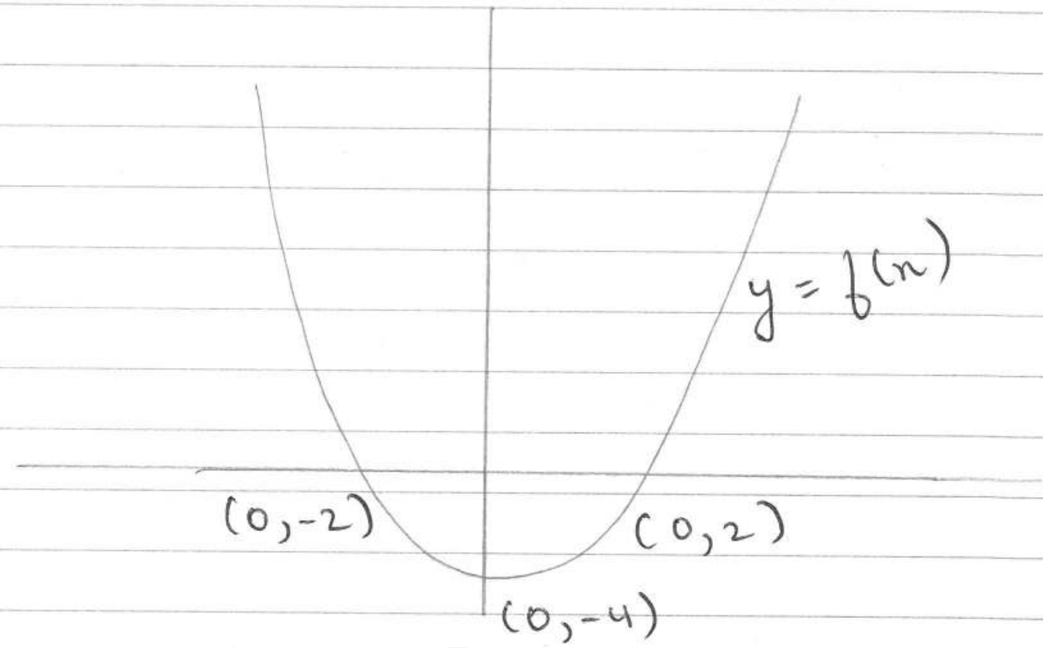
$$\frac{y+8}{4} = 1 + \frac{x^2}{4}$$

$$y + 8 = 4 + x^2$$

$$y = x^2 - 4$$

8.2 Using trigonometric identities

bii]



3c i)

$x = 10\sin t$

$\frac{x}{10} = \sin t$ —— ①

$y = \frac{1}{5}\operatorname{cosec}^2 t$

$5y = \operatorname{cosec}^2 t$

$5y = \frac{1}{\sin^2 t}$ —— ②

Sub ① in ②

$5y = \frac{1}{\left(\frac{x}{10}\right)^2}$

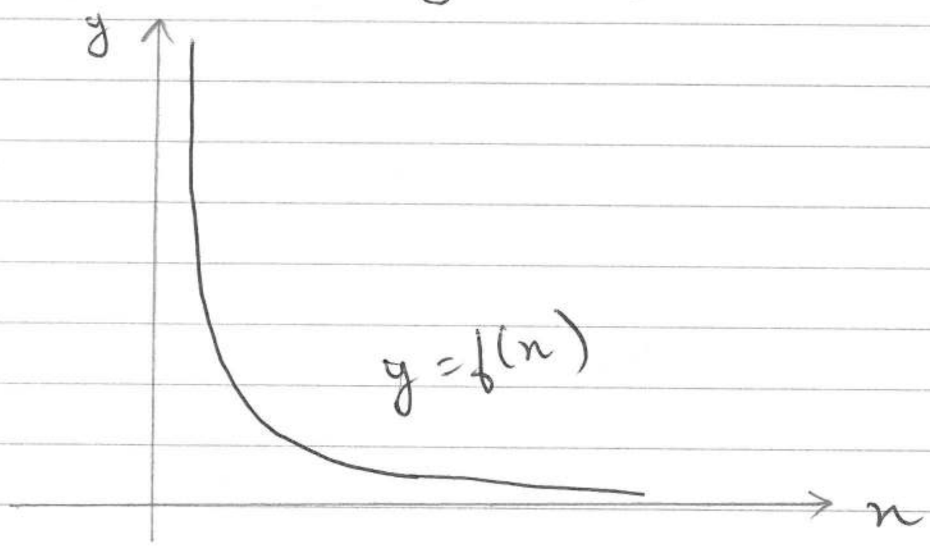
$5y = \frac{1}{\frac{x^2}{100}}$

$5y = \frac{100}{x^2}$

$y = \frac{20}{x^2}$

8.2 Using trigonometric identities

3c ii)



4a) $x = 6\cos t + 5$ $\qquad$ $y = 6\sin t - 2$

$\frac{x-5}{6} = \cos t$ $\qquad$ $\frac{y+2}{6} = \sin t$

Using $\sin^2 t + \cos^2 t = 1$

$$\left(\frac{y+2}{6}\right)^2 + \left(\frac{x-5}{6}\right)^2 = 1$$

$$(x-5)^2 + (y+2)^2 = 36$$

Hence Centre $= (5, -2)$, radius $= 6$

b) Since semicircle is half of a circle

$K = \frac{2\pi}{2}$

So $K = \pi$

5 a) $x = 5\sec^2 2t$ $\qquad$ $y = 2\cot^2 2t$

$\frac{x}{5} = \sec^2 2t$ $\qquad$ $\frac{y}{2} = \cot^2 2t$

$\frac{x}{5} = \frac{1}{\cos^2 2t}$ $\qquad$ $\frac{2}{y} = \tan^2 2t$

$\frac{5}{x} = \cos^2 2t$ — ① $\qquad$ $\frac{2}{y} = \frac{\sin^2 2t}{\cos^2 2t}$

8.2 Using trigonometric identities

5a] Cont.

$$\frac{2}{y} = \frac{(1-\cos^2 2t)}{\cos^2 2t} \quad — ②$$

Sub ① in ②

$$\frac{2}{y} = \frac{\left(1-\frac{5}{x}\right)}{\frac{5}{x}}$$

$$\frac{2}{y} = \frac{\frac{x-5}{\cancel{x}}}{\frac{5}{\cancel{x}}}$$

$$\frac{2}{y} = \frac{x-5}{5}$$

$$10 = xy - 5y$$

$$10 = (x-5)y$$

$$y = \frac{10}{x-5}$$

b] $\sec^2(2t) \geqslant 1$

$x = 5\sec^2(2t) \geqslant t$

As $t = \frac{\pi}{4}$

$$\sec^2\left(\frac{\pi}{2}\right) = \infty$$

Hence $x > 5$

6] $x = 2\sin t$

$\frac{x}{2} = \sin t$

$\cos^2 t = 1 - \sin^2 t$

$\cos t = \sqrt{1-\sin^2 t}$

8.2 Using trigonometric identities

6] Contd

$$y = \cos\left(t + \frac{\pi}{3}\right)$$

$$y = \cos t \cos\frac{\pi}{3} - \sin t \sin\frac{\pi}{3}$$

$$y = \cos t\left(\frac{1}{2}\right) - \sin t\left(\frac{\sqrt{3}}{2}\right)$$

$$y = \frac{1}{2}\cos t - \frac{\sqrt{3}}{2}\sin t$$

Using $\cos t = \sqrt{1-\sin^2 t}$

$$y = \frac{1}{2}\sqrt{1-\sin^2 t} - \frac{\sqrt{3}}{2}\sin t$$

$$y = \frac{1}{2}\sqrt{1-\frac{x^2}{4}} - \frac{\sqrt{3}}{2}\left(\frac{x}{2}\right)$$

$$y = \frac{1}{2}\sqrt{1-\frac{x^2}{4}} - \frac{\sqrt{3}x}{4}$$

$$y = \frac{1}{2}\sqrt{\frac{4-x^2}{4}} - \frac{\sqrt{3}x}{4}$$

$$y = \frac{1}{2}\frac{\sqrt{4-x^2}}{2} - \frac{\sqrt{3}x}{4}$$

$$y = \frac{1}{4}\sqrt{4-x^2} - \frac{\sqrt{3}}{4}x$$

So $y = \frac{1}{4}\left(\sqrt{4-x^2} - \sqrt{3}x\right)$

Hence proved.

Domain : $-2 < x < 2$

8.2 Using trigonometric identities

7a)

$x = \cot^2 t + 3$ $\qquad\qquad$ $y = 3\cos t$

$x - 3 = \cot^2 t$ $\qquad\qquad$ $\frac{y}{3} = \cos t$ — ①

$\frac{1}{x-3} = \frac{\sin^2 t}{\cos^2 t}$

$\frac{1}{x-3} = \frac{1-\cos^2 t}{\cos^2 t}$ — ②

Sub ① in ②

$\frac{1}{x-3} = \frac{1-\left(\frac{y}{3}\right)^2}{\left(\frac{y}{3}\right)^2}$

$\frac{1}{x-3} = \frac{1-\frac{y^2}{9}}{\frac{y^2}{9}}$

$\frac{1}{x-3} = \frac{\frac{9-y^2}{\cancel{9}}}{\frac{y^2}{\cancel{9}}}$

$\frac{1}{x-3} = \frac{9-y^2}{y^2}$

$y^2 = (9-y^2)(x-3)$

$y^2 = 9x - 27 - xy^2 + 3y^2$

$9x - 27 - xy^2 + 2y^2 = 0$

$y^2(2-x) = -9x + 27$

$y^2 = \frac{-9x+27}{2-x}$

$y^2 = 9\frac{(-x+3)}{(2-x)}$

$y^2 = 9\left(\frac{x-3}{x-2}\right)$

7 a) Cont.

$$y = \sqrt{9\left(\frac{x-3}{x-2}\right)}$$

$$y = 3\sqrt{\frac{x-3}{x-2}}$$

b) Domain:

$x = \cot^2 t + 3$

when $t = \frac{\pi}{3}$

$x = 3$

So domain $= x > 3$

Range:

$y = 3\cos t$

when $t = 0$

$y = 3$

when $t = \frac{\pi}{2}$

$y = 0$

$0 < y < 3$