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7.4: Solving trigonometric equations

① a) $\sin\left(x+\frac{\pi}{3}\right)+4\sin x=0$ $\quad \sin(A+B)=\sin A\cos B+\sin B\cos A$

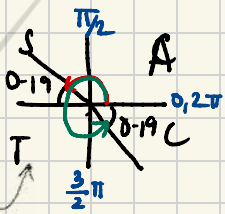
$0\le x\le 2\pi$

$\Rightarrow \sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}+4\sin x=0$

$\Rightarrow \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x+4\sin x=0$

$\Rightarrow \frac{9}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=0 \Rightarrow \frac{9}{2}\sin x=-\frac{\sqrt{3}}{2}\cos x$

$\Rightarrow \tan x=-\frac{\sqrt{3}}{9}$ $\quad x=\arctan\left[-\frac{\sqrt{3}}{9}\right]$ $\quad x=-0.19\pi$



$x=\pi-0.19,\ 2\pi-0.19$

$x=3.0,\ 6.1$ (1dp)

↳ tan is negative in sin and cos

b) $\cos\left(\frac{\pi}{6}+x\right)=\sin x$ $\quad \cos(A+B)=\cos A\cos B-\sin A\sin B$

$\Rightarrow \cos\frac{\pi}{6}\cos x-\sin\frac{\pi}{6}\sin x=\sin x$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x=\sin x$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x=\frac{3}{2}\sin x$

$3\sin x=\sqrt{3}\cos x$

$\tan x=\frac{\sqrt{3}}{3}$ $\quad x=\frac{\pi}{6},\ \pi+\frac{\pi}{6}$

$x=\frac{\pi}{6},\ \frac{7\pi}{6} \Rightarrow x=0.5,\ 3.7$ (1dp)

c) $\cos\left(x+\frac{\pi}{4}\right)+\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{2}$

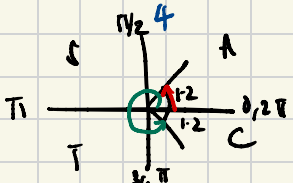
$\Rightarrow \cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}+\sin x\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos x=\frac{1}{2}$

$\Rightarrow \cos x\left(\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\cos x=\frac{1}{2}$

$\Rightarrow \sqrt{2}\cos x=\frac{1}{2}$ $\quad \cos x=\frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $\quad \cos x=\frac{\sqrt{2}}{4}$

$x=\arccos\left(\frac{\sqrt{2}}{4}\right) \Rightarrow x=1.2,\ 2\pi-1.2$

$x=1.2,\ 5.1$ (1dp)



② $0 \le \theta \le 180°$

$\cos 2\theta = 2\cos^2\theta - 1$

a) $1 + \cos 2\theta = 7\cos^2\theta - 1$

$\Rightarrow 1 + 2\cos^2\theta - 1 - 7\cos^2\theta + 1 = 0$

$\Rightarrow 1 - 5\cos^2\theta = 0 \quad (5\cos + \sqrt{5})(5\cos - \sqrt{5}) = 0$

$\Rightarrow \cos\theta = \pm \frac{\sqrt{5}}{5}$

$\cos\theta = +\frac{\sqrt{5}}{5} \quad \theta = 63.4° \qquad \cos\theta = \frac{\sqrt{5}}{5} \Rightarrow \theta = 116.6°$

$\theta = 63.4°, 116.6° \ (1dp)$

b) $\sin 2\theta = \frac{1}{4}\sin\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

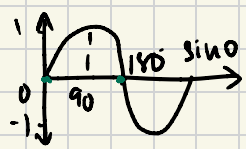
$\Rightarrow 2\sin\theta\cos\theta = \frac{1}{4}\sin\theta \quad \Rightarrow 8\sin\theta\cos\theta = \sin\theta$

$\Rightarrow 8\sin\theta\cos\theta - \sin\theta = 0 \quad \Rightarrow \sin\theta(8\cos - 1) = 0$

$\sin\theta = 0 \qquad \cos\theta = \frac{1}{8}$

$\theta = 0°, 180° \qquad \theta = 82.8°$

$\Rightarrow \theta = 0, 82.8°, 180°$



c) $\tan\theta \tan 2\theta = 3$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\Rightarrow \frac{\tan\theta \times 2\tan\theta}{1 - \tan^2\theta} = 3 \quad \Rightarrow \frac{2\tan^2\theta}{1 - \tan^2\theta} = 3$

$\Rightarrow 2\tan^2\theta = 3 - 3\tan^2\theta \quad \Rightarrow 5\tan^2\theta = 3$

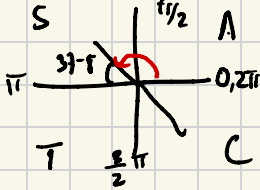
$\tan^2\theta = \frac{3}{5} \qquad \tan\theta = \pm\sqrt{\frac{3}{5}}$

$\tan\theta = \frac{\sqrt{3}}{5}$ $\theta = 37.8$ $\tan\theta = -\frac{\sqrt{3}}{5}$

$\theta = \pi - 37.8$

$\theta = 142.2$

$\Rightarrow \theta = 37.8, 142.2$ (1dp)



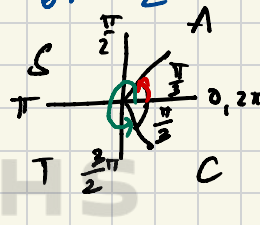
(3) $\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}(\sqrt{3}\cos x + \sin x)$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

a) $\Rightarrow \cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6} \Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x \Rightarrow \frac{1}{2}(\sqrt{3}\cos x + \sin x)$

b) $\sqrt{3}\cos x + \sin x = 1 \Rightarrow 2\cos\left(x - \frac{\pi}{6}\right) = 1 \Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

$\Rightarrow x - \frac{\pi}{6} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\Rightarrow x - \frac{\pi}{6} = \frac{\pi}{3}, \frac{5}{3}\pi$ $x = \left(\frac{\pi}{3} + \frac{\pi}{6}\right), \left(\frac{5\pi}{3} + \frac{\pi}{6}\right)$



$\Rightarrow x = \frac{\pi}{2}, \frac{11}{6}\pi$

(4) a) $\sin(x - 45^\circ) = \cos x$ $\sin(A-B) = \sin A\cos B - \sin B\cos A$

$\Rightarrow \sin x\cos 45^\circ - \sin 45^\circ\cos x = \cos x$

$\Rightarrow \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = \cos x \Rightarrow \frac{\sqrt{2}}{2}\sin x = \frac{2+\sqrt{2}}{2}\cos x$

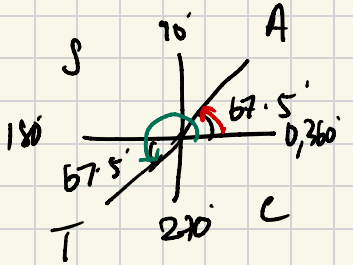
$\Rightarrow \frac{\sin x}{\cos x} = 1 + \sqrt{2}$ $\tan x = 1 + \sqrt{2}$ $a = 1$ $b = \sqrt{2}$

$1 + \sqrt{6}$

$\tan x = 1 + \sqrt{2}$ $x = \arctan(1 + \sqrt{2})$

$x = 67.5^\circ, 180^\circ + 67.5^\circ$

$x = 67.5^\circ, 247.5^\circ$



⑤ a) $2\sin\left[x-\frac{\pi}{6}\right] = \sin\left[x+\frac{\pi}{2}\right]$

$\Rightarrow 2\left[\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6}\right] = \sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2}$

$\Rightarrow 2\sin x\cos\frac{\pi}{6} - 2\cos x\sin\frac{\pi}{6} = \sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2}$

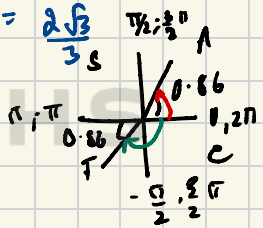
$\Rightarrow \sqrt{3}\sin x - \cos x = \cos x$

$\Rightarrow \sqrt{3}\sin x = 2\cos x \quad \Rightarrow \sqrt{3}\sin x - 2\cos x = 0$

b) $\sqrt{3}\sin x - 2\cos x = 0 \quad \sqrt{3}\sin x = 2\cos x \quad \tan x = \frac{2\sqrt{3}}{3}$

$x = \arctan\left[\frac{2\sqrt{3}}{3}\right] \Rightarrow x = 0.86, -\pi + 0.86$

$x = -2.28, 0.86$



⑥ a) $\tan 2x - \tan x = 0 \quad ; \quad \tan x + \tan^3 x = 0$ $\tan 2x =$ formula booklet

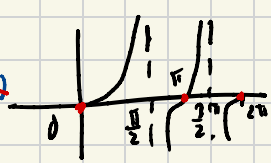
$\Rightarrow \frac{2\tan x}{1-\tan^2 x} - \tan x = 0 \qquad 2\tan x - \tan x(1-\tan^2 x) = 0$

$\Rightarrow 2\tan x - \tan x + \tan^3 x = 0 \Rightarrow \tan x + \tan^3 x = 0$

b) $\tan x(1+\tan^2 x) = 0 \qquad \tan x = 0$

$x = 0, \pi, 2\pi$

$\tan^2 x = -1$



⑦ $2\sin^2 x - \cos 2x - \cos x = 0$

$\Rightarrow 2(1-\cos^2 x) - 2\cos^2 x - 1 - \cos x = 0$

$\Rightarrow 2 - 2\cos^2 x - 2\cos^2 x + 1 - \cos x = 0$

$\Rightarrow -4\cos^2 x - \cos x + 3 = 0$

$\Rightarrow 4\cos^2 x + \cos x - 3 = 0$

b) $4\cos^2 x - \cos x - 1 = 0 \quad (4\cos x - 3)(\cos x + 1) = 0$

$\cos x = \frac{3}{4}$ $\quad$ $\cos x = -1$

$x = 0.72, -0.72$ $\quad$ $x = -\pi, \pi$

$x = -\pi, -0.72, 0.72, \pi$

(8) $\cos^2 x - 2\sin^2 x - \sin 2x = 1$

$\Rightarrow \cos^2 x - 2\sin^2 x - 2\sin x \cos x = 1$

$\Rightarrow \cos^2 x - \sin^2 x - 2\sin^2 x - 2\sin x \cos x - 1 = 0$

$\Rightarrow -3\sin^2 x - 2\sin x \cos x = 0$

$\Rightarrow 3\sin^2 x + 2\sin x \cos x = 0$

$\Rightarrow \sin x (3\sin x + 2\cos x) = 0$

$\sin x = 0 \quad x = 0, -\pi$

$3\sin x = -2\cos x \quad \tan x = -\frac{2}{3}$

$x = -0.59, (\pi - 0.59 = 2.55)$

$x = -\pi, -0.59, 0, 2.55$