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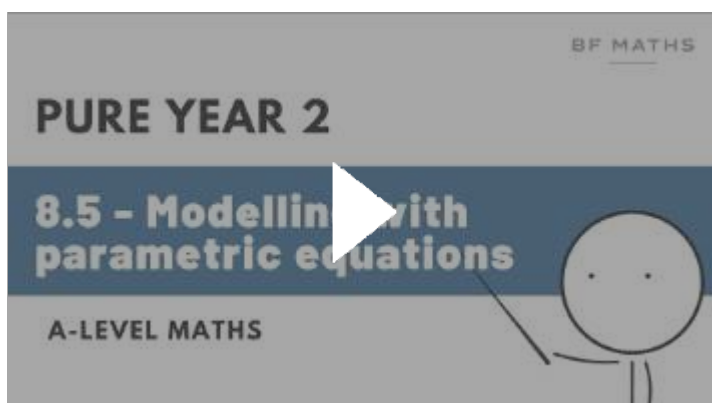
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If you need help on this chapter:

[A-Level Maths | Pure Year 2 | 8.5 - Modelling with parametric equations Walkthrough | Edexcel](#)



8.5 - Modelling with parametric equations

1a) When $y = 450$,

$$450 = 2.7t$$

$$t = \underline{\underline{\frac{500}{3} \text{ secs}}}$$

1b) $x = 3.6t$

$$t = \frac{x}{3.6}$$

Sub into y

$$y = 2.7 \left(\frac{x}{3.6} \right)$$

$$y = \frac{3}{4}x$$

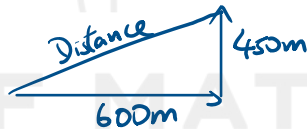
⇒ The equation is linear

∴ The motion is a straight line.

1c) When $t = \frac{500}{3}$,

$$x = 3.6 \left(\frac{500}{3} \right) = 600\text{m}$$

$$\begin{aligned} \therefore \text{Distance} &= \sqrt{600^2 + 450^2} \\ &= \underline{\underline{750\text{m}}} \end{aligned}$$



1d) $\text{Speed} = \frac{\text{Dist}}{\text{Time}} = \frac{750}{\frac{500}{3}} = \underline{\underline{4.5 \text{ ms}^{-1}}}$

1e.i) The path of the skier is unlikely to be a perfect straight line.

1e.ii) The speed of ski lift is unlikely to be constant.
(eg. when arriving at the terminal, it slows down gradually)

8.5 - Modelling with parametric equations

2a) When $t=0$, $y=16-0-1=15$
 \therefore Initial height = 15m

2b) When $t=\frac{17}{8}$, $y=16-7.6\left(\frac{17}{8}\right)-6s\left(8\times\frac{17}{8}\right)$
 $=\underline{\underline{0.125m}}$

2c) When $t=\frac{17}{8}$, $x=8\left(\frac{17}{8}\right)=\underline{\underline{17m}}$

3a) When $x=33$, $33=9\sqrt{2}t$
 $t=\frac{33}{9\sqrt{2}}=\underline{\underline{2.59 \text{ secs}}}$

3b) When $t=2.59$, $y=0$ ← ball hits the ground

$$0 = -k(2.59)^2 + 9\sqrt{2}(2.59)$$

$$k = \frac{9\sqrt{2}(2.59)}{(2.59)^2} = \underline{\underline{4.91}}$$

3c) $x=9\sqrt{2}t$, $y=-4.91t^2+9\sqrt{2}t$
↓

$$t = \frac{x}{9\sqrt{2}} \rightarrow y = -4.91\left(\frac{x}{9\sqrt{2}}\right)^2 + 9\sqrt{2}\left(\frac{x}{9\sqrt{2}}\right)$$

$$\underline{\underline{y = -\frac{4.91}{16200}x^2 + x}}$$

8.5 - Modelling with parametric equations

4a) $x = 5.6t$, $y = -4.9t^2 + 2.1t$

When $y=0$, $0 = -4.9t^2 + 2.1t$

$$0 = t(-4.9t + 2.1)$$

$$t=0 \text{ or } -4.9t + 2.1 = 0$$

$$t = \frac{2.1}{4.9} = \frac{3}{7}$$

\therefore Takes $\frac{3}{7}$ secs to complete a single jump.

4b) When $t = \frac{3}{7}$, $x = 5.6\left(\frac{3}{7}\right) = \underline{\underline{2.4m}}$

4c) When $y = 0.2$, $0.2 = -4.9t^2 + 2.1t$

$$0 = -4.9t^2 + 2.1t - 0.2$$

$$t = \frac{1}{7} \text{ or } \frac{2}{7}$$

When $t = \frac{1}{7}$, $x = 5.6\left(\frac{1}{7}\right) = 0.8m$

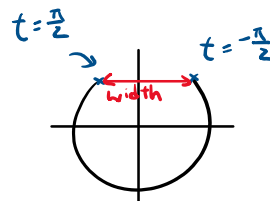
$t = \frac{2}{7}$, $x = 5.6\left(\frac{2}{7}\right) = 1.6m$

$$\therefore \underline{\underline{0.8 < x < 1.6m}}$$

5a) When $t = -\frac{\pi}{2}$, $x = 18\left(-\frac{\pi}{2}\right) - 32\sin\left(-\frac{\pi}{2}\right) = 3.726$

When $t = \frac{\pi}{2}$, $x = 18\left(\frac{\pi}{2}\right) - 32\sin\left(\frac{\pi}{2}\right) = -3.726$

$$\therefore \text{width} = 3.726 - (-3.726) = \underline{\underline{7.45m}}$$



5b) When $y=0$, $0 = 8 - 16\cos t$

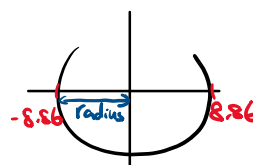
$$\cos t = \frac{8}{16}$$

$$t = \frac{\pi}{3} \text{ or } -\frac{\pi}{3}$$

When $t = \frac{\pi}{3}$, $x = 18\left(\frac{\pi}{3}\right) - 32\sin\left(\frac{\pi}{3}\right) = -8.86$

$t = -\frac{\pi}{3}$, $x = 8.86$

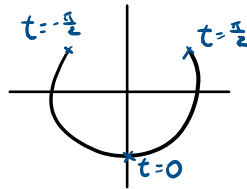
$$\therefore \text{radius} = \frac{8.86 - (-8.86)}{2} = \underline{\underline{8.86cm}}$$



8.5 - Modelling with parametric equations

5c) When $t=0$, $y = 8 - 16\cos 0$
 $= -8$

\therefore depth = 8cm



6a) $x = t + 5$, $y = 5t - 2$
 \downarrow

$t = x - 5 \rightarrow y = 5(x - 5) - 2$
 $y = 5x - 27$

6b) $x = \frac{t^2 - 6t + 5}{t + 1}$, $y = t - 2$
 \downarrow
 $\leftarrow t = y + 2$

$x = \frac{(y+2)^2 - 6(y+2) + 5}{y+2+1}$

$x = \frac{y^2 + 4y + 4 - 6y - 12 + 5}{y+3}$

$x(y+3) = y^2 - 2y - 3$

Sub $y = 5x - 27$

$x(5x - 27 + 3) = (5x - 27)^2 - 2(5x - 27) - 3$

$5x^2 - 24x = 25x^2 - 270x + 729 - 10x + 54 - 3$

$0 = 20x^2 - 256x + 780$

$x = 7.8$ or 5

When $x = 7.8$, $y = 5(7.8) - 27 = 12$

$\therefore (7.8, 12)$

6c) Sub $y = 12$ into $y = t - 2$ $\left\{ \right.$ Sub $y = 12$ into $y = 5t - 2$
 $t = 14$ $\left. \right\}$ $t = \frac{14}{5}$

The values of t don't match
 so particles won't collide.

8.5 - Modelling with parametric equations

7a) When $t=0$, $x = 400 \cos\left(\frac{1}{5} \times 0 - \frac{\pi}{3}\right)$
 $= 391.26$

$$y = 400 \sin^2(0) - 200 \sin(0) = 0$$

\therefore distance = 391.3 m

7b) due north = on the y-axis ($x=0$)

$$0 = 400 \cos\left(\frac{1}{5}\left(t - \frac{\pi}{3}\right)\right)$$

$$0 = \cos\left(\frac{1}{5}\left(t - \frac{\pi}{3}\right)\right)$$

$$\frac{1}{5}\left(t - \frac{\pi}{3}\right) = \frac{\pi}{2} \quad \text{or} \quad \frac{3\pi}{2}$$

$$t = \frac{17\pi}{6} \quad \text{or} \quad \frac{47\pi}{6}$$

$\downarrow \times 5 + \frac{\pi}{3}$

When $t = \frac{17\pi}{6}$, $y = 187 \text{ m}$

$t = \frac{47\pi}{6}$, $y = 578 \text{ m}$

7c) Complete one loop = return to the starting point

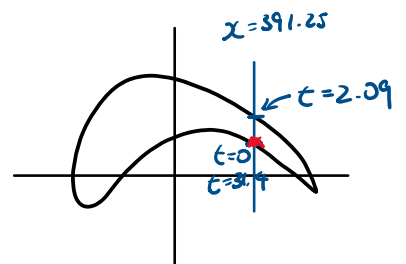
When $x = 391.3$, $391.259 = 400 \cos\left(\frac{1}{5}\left(t - \frac{\pi}{3}\right)\right)$
 (from (a))

$$\cos\left(\frac{1}{5}\left(t - \frac{\pi}{3}\right)\right) = 0.9781475$$

$$\frac{1}{5}\left(t - \frac{\pi}{3}\right) = 0.209, 6.07$$

$$t = 2.09, \quad \textcircled{31.4}$$

Takes 31 mins to complete one loop.



8.5 - Modelling with parametric equations

8a) $y = 20 \sin t$

maximum value when $\sin t = 1$

$$\therefore y = 20(1) = \underline{\underline{20\text{m}}}$$

8b) When $y=0$, $0 = 20 \sin t$

$$\sin t = 0$$

$$t = 0 \text{ or } \pi$$

$$\therefore k = \pi$$

$$\text{gradient} = \frac{\text{d}y}{\text{d}x}$$

when $y=20$, $20 = 20 \sin t$

$$\sin t = 1$$

$$t = \frac{\pi}{2}$$

when $t = \frac{\pi}{2}$, $x = \left(\frac{\pi}{2}\right)^2 + 16\left(\frac{\pi}{2}\right) = 27.6 \leftarrow \text{point A}$

when $t = \pi$, $x = \pi^2 + 16(\pi) = 60.135 \leftarrow \text{point B}$

$$\text{gradient} = - \frac{20}{60.135 - 27.6} = \underline{\underline{-0.615}}$$

