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# 7.3: Double angle formulae

(1) a) $2\sin\frac{\pi}{3}\cos\frac{\pi}{3} = \sin\frac{2\pi}{3}$ $\quad \sin 2A = 2\sin A\cos A$

b) $4\cos^2 7^\circ - 2 \Rightarrow 2(2\cos^2 7^\circ - 1)$ $\quad \cos 2A = 2\cos^2 A - 1$

$\Rightarrow 2\cos 2A = 2\cos 14^\circ$

c) $\dfrac{1-\tan^2 13.5^\circ}{2\tan 13.5^\circ} = \dfrac{1}{\frac{2\tan 13.5}{1-\tan^2 13.5}} = \dfrac{1}{\tan 2(13.5^\circ)}$

$\dfrac{2\tan A}{1-\tan^2 A} = \tan 2A$ $\quad \Rightarrow \dfrac{1}{\tan 27} = \cot 27^\circ$

(2) a) $6\sin 5\theta\cos 5\theta \Rightarrow 3(2\sin 5\theta\cos 5\theta)$

$\sin 2A = 2\sin A\cos A$ $\quad \Rightarrow 3\sin 2A = 3\sin 10\theta$

b) $\tan\theta(1+\cos 2\theta) = \tan\theta + \tan\theta\cos 2\theta$

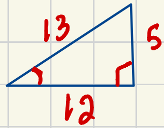
$\Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}(\cos 2\theta) = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}(1-2\sin^2\theta)$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin\theta - 2\sin^3\theta}{\cos\theta} \Rightarrow \dfrac{2\sin\theta - 2\sin^3\theta}{\cos\theta} = \dfrac{2\sin\theta(1-\sin^2\theta)}{\cos\theta}$

$\Rightarrow \dfrac{2\sin\theta\,\cancel{\cos^2\theta}^{\cos\theta}}{\cancel{\cos\theta}} = 2\sin\theta\cos\theta = \sin 2A$ $\quad \sin 2A = 2\sin A\cos A$

c) $\dfrac{\sin 2\theta}{1-\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)} \Rightarrow \dfrac{2\cancel{\sin\theta}\cos\theta}{\cancel{1}-\cancel{1}+2\sin^{\cancel{2}}\theta} = \dfrac{\cancel{2}\cos\theta}{\cancel{2}\sin\theta} = \cot\theta$

③ $\sin A = \frac{5}{13}$ $\cos A = -\frac{12}{13}$ $\tan A = -\frac{5}{12}$




a) $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\left[-\frac{5}{12}\right]}{1-\left[-\frac{5}{12}\right]^2} = \frac{-120}{119}$

b) $\cos 2A = 2\cos^2 A - 1 = 2\left(-\frac{12}{13}\right)^2 - 1 = \frac{119}{169}$

c) $\operatorname{cosec} 2A = \frac{1}{\sin 2A} = \frac{1}{2\sin A\cos A} = \frac{1}{2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)} = -\frac{169}{120}$

④ a) $x = 5\sin\theta$ $y = 2 - 10\cos 2\theta$ $\cos 2\theta = 1 - 2\sin^2\theta$

$\boxed{\frac{x}{5} = \sin\theta}$

$y = 2 - 10(1 - 2\sin^2\theta)$

$\Rightarrow y = 2 - 10 + 20\sin^2\theta$ substitute $\boxed{\sin\theta = \frac{x}{5}}$

$\Rightarrow y = -8 + 20\left[\frac{x}{5}\right]^2$

$y = -8 + \frac{\overset{4}{\cancel{20}}\, x^2}{\underset{5}{\cancel{25}}}$ $\Rightarrow$ $y = \frac{4}{5}x^2 - 8$

b) $x = \cos 2\theta$ $y = 4\sec\theta$

$x = 2\cos^2\theta - 1$

$\frac{x+1}{2} = \cos^2\theta$

$\frac{2}{x+1} = \sec^2\theta$

$\sec\theta = \sqrt{\frac{2}{x+1}}$

$y = 4\left[\frac{\sqrt{2}}{\sqrt{x+1}}\right] = y \Rightarrow \frac{4\sqrt{2}}{\sqrt{x+1}}$

⑤ a) $\underbrace{\cos^4 x - \sin^4 x}_{\text{difference of square}} = \cos 2x$

$$\Rightarrow (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x - \sin^2 x = \cos 2x$

$$\Rightarrow (1)(\cos 2x) = \cos 2x$$

b) $\cos^4 \frac{\pi}{12} - \sin^4 \frac{\pi}{12} = \cos \cancel{2}\left[\frac{\pi}{\cancel{12}_6}\right] = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

⑥ $a = 2\sin x$, $b = 2\cos x$ $c = \cos 2x$ $d = \tan 2x$

a) $c = \cos 2x = 1 - 2\sin^2 x$ — ① $\boxed{\sin x = \frac{a}{2}}$ — ②

Sub eq ② into eq ①

$$c = 1 - 2\left[\frac{a}{2}\right]^2 \Rightarrow 1 - \cancel{2}\left[\frac{a}{\cancel{4}_2}\right] \Rightarrow c = \frac{2 - a^2}{2}$$

$$\Rightarrow c = 1 - \frac{a^2}{2}$$

b) $d = \frac{2ab}{b^2 - a^2}$ $d = \tan 2x = \frac{2\tan x}{1 - \tan^2 x} = \frac{2\left(\frac{\sin x}{\cos x}\right)}{1 - \frac{\sin^2 x}{\cos^2 x}}$

$\sin x = \frac{a}{2}$

$\cos x = \frac{b}{2}$

$c = \cos 2x = \cos^2 x - \sin^2 x$

$$\Rightarrow \frac{2\left(\frac{\sin x}{\cos x}\right)}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$$

$$\Rightarrow \frac{2\sin x}{\cancel{\cos x}} \times \frac{\cancel{\cos^2 x}\ \cos x}{\cos^2 x - \sin^2 x}$$

$$\Rightarrow \frac{2\left(\frac{a}{2}\right)\left(\frac{b}{2}\right)}{\left(\frac{b}{2}\right)^2 - \left(\frac{a}{2}\right)^2} = \frac{\cancel{2}\left(\frac{ab}{\cancel{4}_2}\right)}{\frac{b^2 - a^2}{4}} = \frac{ab}{\cancel{2}} \times \frac{\cancel{4}^2}{b^2 - a^2}$$

$$= \frac{2ab}{b^2 - a^2}$$

⑦ $\cos\theta = \frac{24}{25}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

the $\frac{\theta}{2}$ is positive because $\theta$ is halfed. It would lie in 2nd quadrant where sin is "+"

$\sin\theta = \sqrt{\frac{1-\cos\theta}{2}}$

$\Rightarrow \frac{\sqrt{1-\left(\frac{24}{25}\right)}}{2}$

$\Rightarrow \frac{\sqrt{2}}{10}$