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7.2 : Solution bank

① a) $\sin\frac{\pi}{12}\cos\frac{\pi}{4} + \cos\frac{\pi}{12}\sin\frac{\pi}{4}$

$$\Rightarrow \left[\frac{-1+\sqrt{3}}{4}\right] + \left[\frac{1+\sqrt{3}}{4}\right] = \frac{2\sqrt{3}}{\cancel{4}\,2} = \boxed{\frac{\sqrt{3}}{2}}$$

b) $\cos 96^\circ \cos 51^\circ + \sin 96^\circ \sin 51^\circ = \cos(96^\circ - 51^\circ) = \cos 45^\circ = \boxed{\frac{\sqrt{2}}{2}}$

c) $$\frac{\tan\frac{\pi}{18} + \tan\frac{\pi}{9}}{1 - \tan\frac{\pi}{18}\tan\frac{\pi}{9}} = \tan\left[\frac{\pi}{18} + \frac{\pi}{9}\right] = \boxed{\frac{\sqrt{3}}{3}}$$

② a) $\sin(A+B) = \sin A\cos B + \sin B\cos A$

$$\Rightarrow \sin(135^\circ) = \sin(45^\circ + 90^\circ)$$

$$\Rightarrow \sin 45^\circ \cos 90^\circ + \cos 45^\circ \sin 90^\circ = \left[\frac{\sqrt{2}}{2}(0)\right] + \left[1\left(\frac{\sqrt{2}}{2}\right)\right]$$

$$\Rightarrow 0 + \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{2}}$$

b) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\Rightarrow \cos(15^\circ) = \cos(45^\circ - 30^\circ)$$

$$\Rightarrow \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

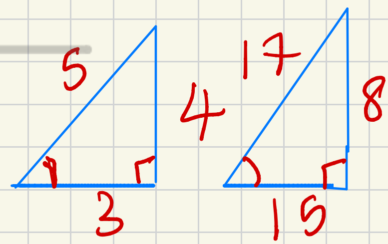
c) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(75^\circ) = \tan(30^\circ + 45^\circ)$

$\Rightarrow \dfrac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = \dfrac{\frac{\sqrt{3}}{3} + 1}{1 - \left(\frac{\sqrt{3}}{3}\right)(1)} = \dfrac{\frac{\sqrt{3}+3}{\cancel{3}}}{\frac{3-\sqrt{3}}{\cancel{3}}}$

$\tan 75^\circ = 2 + \sqrt{3}$

(3) $\cos A = \dfrac{3}{5}$ $\qquad \sin B = \dfrac{8}{17}$



$\sin A = \dfrac{4}{5}$ $\qquad \cos B = \dfrac{15}{17}$

$\tan A = \dfrac{4}{3}$ $\qquad \tan B = \dfrac{8}{15}$

a) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\Rightarrow \dfrac{3}{5}\left(\dfrac{15}{17}\right) - \dfrac{4}{5}\left(\dfrac{8}{17}\right) = \dfrac{13}{85}$

b) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B} = \dfrac{\frac{4}{3}-\frac{8}{15}}{1+\left(\frac{4}{3}\right)\left(\frac{8}{15}\right)}$

$\Rightarrow \tan(A-B) = \boxed{\dfrac{36}{77}}$

c) $\operatorname{cosec}(A+B) = \dfrac{1}{\sin(A+B)} = \dfrac{1}{\sin A\cos B + \sin B \cos A}$

$\Rightarrow \dfrac{1}{\frac{4}{5}\left(\frac{15}{17}\right)+\left(\frac{3}{5}\right)\left(\frac{8}{17}\right)} = \dfrac{1}{\frac{12}{17}+\frac{24}{85}} = \dfrac{85}{84}$

(4) $\cos A = \dfrac{1}{2}$ $\quad A = \arccos\left(\dfrac{1}{2}\right)$ $\quad \boxed{A = \dfrac{\pi}{3}}$

a) $\cos\left[\dfrac{\pi}{6} - A\right] = \cos\left(\dfrac{\pi}{6}-\dfrac{\pi}{3}\right) = \cos(A-B)$

$\Rightarrow \cos\dfrac{\pi}{6}\cos\dfrac{\pi}{3} + \sin\dfrac{\pi}{6}\sin\dfrac{\pi}{3}$

$= \dfrac{\sqrt{3}}{2}\left(\dfrac{1}{2}\right) + \dfrac{\sqrt{3}}{2}\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{3}}{4}+\dfrac{\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

b) $\tan\left[\frac{\pi}{3} + \frac{\pi}{3}\right] = \tan(A+B)$

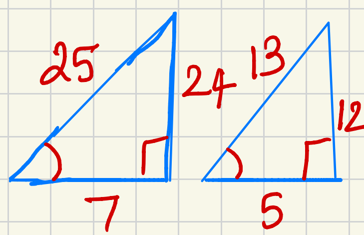
$$\Rightarrow \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{3}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{3}} = \frac{\sqrt{3}+\sqrt{3}}{1-3} = \frac{\cancel{2}\sqrt{3}}{-\cancel{2}} = \boxed{-\sqrt{3}}$$

c) $\sin\left[\frac{\pi}{2} - \frac{\pi}{3}\right] = \sin(A-B)$

$$\sin\frac{\pi}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{\pi}{2} = 1\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2}(0)$$

$$\Rightarrow \frac{1}{2} - 0 = \boxed{\frac{1}{2}}$$

(5) $\sin A = \frac{24}{25}$ $\qquad \cos B = -\frac{5}{13}$



$\cos A = -\frac{7}{25}$ $\qquad \sin B = -\frac{12}{13}$

$\tan A = -\frac{24}{7}$ $\qquad \tan B = \frac{12}{5}$

a) $\sin(A+B) = \sin A\cos B + \sin B\cos A$

$$\Rightarrow \left(\frac{24}{25}\right)\left(\frac{-5}{13}\right) + \left(\frac{-12}{13}\right)\left(\frac{7}{25}\right) = \boxed{-\frac{36}{325}}$$

b) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B} = \dfrac{-\frac{24}{7} - \frac{12}{5}}{1+\left(\frac{-24}{7}\right)\left(\frac{12}{5}\right)}$

$\Rightarrow \tan(A-B) = \dfrac{204}{253}$

c) $\sec(A+B) = \dfrac{1}{\cos(A+B)} = \dfrac{1}{\cos A \cos B - \sin A \sin B}$

$\Rightarrow \dfrac{1}{\left(\frac{-7}{25}\right)\left(\frac{-5}{13}\right) - \left(\frac{24}{25}\right)\left(\frac{-12}{13}\right)} = \dfrac{325}{323}$

6 a) $\tan\left[\dfrac{\pi}{4} - \dfrac{\pi}{6}\right] = \tan(A-B)$

$= \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

$\Rightarrow \dfrac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \tan\dfrac{\pi}{12}$

b) $\tan\dfrac{\pi}{12} = \dfrac{1-\frac{\sqrt{3}}{3}}{1+1\left(\frac{\sqrt{3}}{3}\right)} \Rightarrow 2+\sqrt{3}$

⑦ a) $\sin(x+a) \equiv \sin x \cos a + \cos x \sin a$

$$\sin(105^\circ) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

b) $$\operatorname{cosec} 105^\circ = \frac{1}{\sin 105^\circ} = \frac{1}{\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)}$$

$$\Rightarrow \frac{4}{\sqrt{6}+\sqrt{2}} = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2 - (\sqrt{2})^2}$$

$$\Rightarrow \frac{\cancel{4}(\sqrt{6}-\sqrt{2})}{\cancel{4}} = \sqrt{6}-\sqrt{2}$$