Author: <u>Blinzy Fernandes</u>

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4.1 Expanding $(1+x)^n$

1a) $\dfrac{1}{(1+x)^2} \Longrightarrow (1+x)^{-2}$

$$= 1 + (-2)(x) + \frac{(-2)(-2-1)}{2!}x^2 + \frac{(-2)(-2-1)(-2-2)}{3!}x^3$$

$$= 1 - 2x + 3x^2 - 4x^3$$

b) $(1+x)^{1/2}$

$$= 1 + \left(\tfrac{1}{2}\right)(x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}x^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}x^3$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$$

c) $(1+x)^{4/3}$

$$= 1 + \left(\frac{4}{3}\right)x + \frac{\left(\frac{4}{3}\right)\left(\frac{4}{3}-1\right)}{2!}x^2 + \frac{\left(\frac{4}{3}\right)\left(\frac{4}{3}-1\right)\left(\frac{4}{3}-2\right)}{3!}x^3$$

$$= 1 + \frac{4}{3}x + \frac{2}{9}x^2 - \frac{4}{81}x^3$$

d) $(1+x)^{-\frac{1}{5}}$

$$= 1 + \left(-\frac{1}{5}\right)x + \frac{\left(-\frac{1}{5}\right)\left(-\frac{1}{5}-1\right)}{2!}x^2 + \frac{\left(-\frac{1}{5}\right)\left(-\frac{1}{5}-1\right)\left(-\frac{1}{5}-2\right)}{3!}x^3$$

$$= 1 - \frac{1}{5}x + \frac{3}{25}x^2 - \frac{11}{125}x^3$$

4.1 Expanding $(1+x)^n$

1e) $(1+x)^{-5}$

$$= 1 + (-5)x + \frac{(-5)(-5-1)}{2!}x^2 + \frac{(-5)(-5-1)(-5-2)}{3!}x^3$$

$$= 1 - 5x + 15x^2 - 35x^3$$

f) $(1+x)^{-\frac{5}{2}}$

$$= 1 + \left(\frac{-5}{2}\right)x + \frac{\left(\frac{-5}{2}\right)\left(\frac{-5}{2}-1\right)x^2}{2!} + \frac{\left(\frac{-5}{2}\right)\left(\frac{-5}{2}-1\right)\left(\frac{-5}{2}-2\right)x}{3!}$$

$$= 1 - \frac{5}{2}x + \frac{35}{8}x^2 - \frac{105}{16}x^3$$

2a) i $\frac{1}{(1+2x)^3} = (1+2x)^{-3}$

$$= 1 + (-3)(2x) + \frac{(-3)(-3-1)}{2!}(2x)^2 + \frac{(-3)(-3-1)(-3-2)}{3!}(2$$

$$= 1 - 6x + 24x^2 - 80x^3$$

ii) $1 + 2x < 0$

$x < \frac{-1}{2}$

$|x| < \frac{1}{2}$

b i) $\sqrt{1-x} = (1-x)^{\frac{1}{2}}$

$$= 1 + \left(\tfrac{1}{2}\right)(-x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(-x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)(-x)^3}{3!}$$

4.1 Expanding $(1+x)^n$

2bi] Cont.

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$$

ii] $1 - x < 0$

$-x < -1$

$|x| < 1$

ci] $\left(1 + \frac{x}{2}\right)^{2/3}$

$$= 1 + \left(\frac{2}{3}\right)\left(\frac{x}{2}\right) + \frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)}{2!}\left(\frac{x}{2}\right)^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\left(\frac{2}{3}-2\right)}{3!}\left(\frac{x}{2}\right)^3$$

$$= 1 + \frac{1}{3}x - \frac{1}{36}x^2 + \frac{1}{162}x^3$$

ii] $1 + \frac{x}{2} < 0$

$\frac{x}{2} < -1$

$|x| < 2$

d]i $(1-5x)^{-4}$

$$= 1 + (-4)(-5x) + \frac{(-4)(-4-1)(-5x)^2}{2!} + \frac{(-4)(-4-1)(-4-2)}{3!}(-5x)^3$$

$$= 1 + 20x + 250x^2 + 2500x^3$$

4.1 Expanding $(1+x)^n$

2 d ii] $1 - 5x < 0$

$-5x < -1$

$|x| < \frac{1}{5}$

e i] $\frac{1}{\sqrt[3]{1-2x}} = (1-2x)^{-\frac{1}{3}}$

$= 1 + \left(\frac{-1}{3}\right)(-2x) + \frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)}{2!}(-2x)^2 + \frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)\left(\frac{-1}{3}-2\right)}{3!}(-2x)^3$

$= 1 + \frac{2}{3}x + \frac{8}{9}x^2 + \frac{112}{81}x^3$

ii] $1 - 2x < 0$

$-2x < -1$

$|x| < \frac{1}{2}$

f i] $\left(1 - \frac{x}{3}\right)^{-2}$

$= 1 + (-2)\left(-\frac{x}{3}\right) + \frac{(-2)(-2-1)}{2!}\left(-\frac{x}{3}\right)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}\left(-\frac{x}{3}\right)^3$

$= 1 + \frac{2}{3}x + \frac{1}{3}x^2 + \frac{4}{3}x^3$

ii] $1 - \frac{x}{3} < 0$

$-\frac{x}{3} < -1 \longrightarrow |x| < 3$

4.1 Expanding $(1+x)^n$

3a] $\sqrt{1-4x} = (1-4x)^{1/2}$

$$= 1 + \left(\tfrac{1}{2}\right)(-4x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(-4x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-4x)^3$$

$$= 1 - 2x - 2x^2 - 4x^3$$

b] When $x = 0.01$

$1 - 2(0.01) - 2(0.01)^2 - 4(0.01)^3$

$= 0.979796$ — ①

When $x = 0.01$

$\sqrt{1-4x} \Rightarrow \sqrt{1-4(0.01)} \Rightarrow \frac{2}{5}\sqrt{6}$ — ②

Equate ① and ② together.

$\frac{2}{5}\sqrt{6} = 0.979796$

$\sqrt{6} \approx 2.44949$

c] $\sqrt{1-4x}$ is valid for $|x| < \frac{1}{4}$ and

$0.01 < \frac{1}{4}$

Hence, it is valid.

4a] $f(x) = \dfrac{4+x}{\sqrt{1+3x}} \Longrightarrow (4+x)(1+3x)^{-\frac{1}{2}}$

$$\Rightarrow (4+x)\left(1 + \left(-\tfrac{1}{2}\right)(3x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(3x)^2\right)$$

$$= (4+x)\left(1 - \frac{3}{2}x + \frac{27}{8}x^2\right)$$

4.1 Expanding $(1+x)^n$

4a] Cont.

$= 4 - 6x - \frac{27}{2}x^2 + x - \frac{3}{2}x^2$

$= 4 - 5x + 12x^2$

b] $1 + 3x < 0$

$x < -\frac{1}{3}$

$|x| < \frac{1}{3}$

5a] $(1 + Kx)^{-5}$

$= 1 + (-5)(Kx) + \frac{(-5)(-5-1)}{2!}(Kx)^2$

$= 1 - 5Kx + 15K^2x^2$

Equating the x term

$= \cancel{-}5K\cancel{x} = \cancel{-}8\cancel{x}$

$K = \frac{8}{5}$

b] $15K^2\cancel{x^2} = A\cancel{x^2}$

$\Rightarrow 15\left(\frac{8}{5}\right)^2 = A$

$A = \frac{192}{5}$

4.1 Expanding $(1+x)^n$

6a] $f(x) = \dfrac{1-x}{1+3x} = (1-x)(1+3x)^{-1}$

$= (1-x)\left(1 + (-1)(3x) + \dfrac{(-1)(-1-1)}{2!}(3x)^2 + \dfrac{(-1)(-1-1)(-1-2)(3x)^3}{3!}\right.$

$= (1-x)(1-3x+9x^2-27x^3)$

$= (1-3x+9x^2-27x^3-x+3x^2-9x^3)$

$= 1-4x+12x^2-36x^3$

b] $1+3x < 0$

$|x| < \dfrac{1}{3}$

7a] $g(x) = \dfrac{5}{1+4x} - \dfrac{3}{1-2x}$

$\Rightarrow 5(1+4x)^{-1} - 3(1-2x)^{-1}$

$= 5\left(1 + (-1)(4x) + \dfrac{(-1)(-1-1)}{2!}(4x)^2\right)$

$= 5(1-4x+16x^2)$

$= 5 - 20 + 80x^2$ — ①

$\Rightarrow -3\left(1 + (-1)(-2x) + \dfrac{(-1)(-1-1)}{2!}(-2x)^2\right)$

$= -3(1+2x+4x^2)$

$= -3-6x-12x^2$ — ②

Equate ① and ② together

$= 5-20x+80x^2-3-6x-12x^2$

$= 2-26x+68x^2$

4.1 Equating $(1+x)^n$

7b] $1 + 4x < 0$

$|x| < \frac{1}{4}$

c] $g\left(\frac{1}{100}\right) = 2 - 26\left(\frac{1}{100}\right) + 68\left(\frac{1}{100}\right)^2$

$= 1.7468$

8a] $\sqrt{1-6x} = (1-6x)^{1/2}$

$= 1 + \left(\frac{1}{2}\right)(-6x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)(-6x)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)(-6x)^3}{3!}$

$= 1 - 3x - \frac{9}{2}x^2 - \frac{27}{2}x^3$

b] Sub $x = \frac{1}{300}$ in $\sqrt{1-6x}$

$= \sqrt{1 - 6\left(\frac{1}{300}\right)} = \frac{7\sqrt{2}}{10}$

Sub $x = \frac{1}{300}$ into expansion

$1 - 3\left(\frac{1}{300}\right) - \frac{9}{2}\left(\frac{1}{300}\right)^2 - \frac{27}{2}\left(\frac{1}{300}\right)^3$

$= 0.9899495$

Equate them together

$\frac{7\sqrt{2}}{10} = 0.9899495$

$\sqrt{2} \approx 1.41421$

4.1 Equating $(1+x)^n$

9a) $\sqrt{\dfrac{1-5x}{1+x}} \Rightarrow \left(\dfrac{1-5x}{1+x}\right)^{1/2} \Rightarrow \dfrac{(1-5x)^{1/2}}{(1+x)^{1/2}}$

$\Longrightarrow (1-5x)^{1/2}(1+x)^{-1/2}$

$\Rightarrow (1-5x)^{1/2} = 1 + \left(\frac{1}{2}\right)(-5x) + \dfrac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!} \times (-5x)^2$

$\Rightarrow \quad 1 \cancel{+} - \frac{5}{2}x \; - \cancel{\frac{25}{8}x^2} \; - \frac{25}{8}x^2$

$(1+x)^{1/2} \Rightarrow 1 + \left(-\frac{1}{2}\right)(x) + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(x)^2$

$\Rightarrow 1 - \frac{1}{2}x + \frac{3}{8}x^2$

Equating them together

$= \left(1 - \frac{5}{2}x - \frac{25}{8}x^2\right)\left(1 - \frac{1}{2}x + \frac{3}{8}x^2\right)$

$= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{2}x + \frac{5}{4}x^2 - \frac{25}{8}x^2$

$= 1 - 3x - \frac{3}{2}x^2$

b) Sub $x = \frac{1}{8}$

$= \sqrt{\dfrac{1-5\left(\frac{1}{8}\right)}{1+1/8}} \approx 1 - 3\left(\frac{1}{8}\right) - \frac{3}{2}\left(\frac{1}{8}\right)^2$

$= \sqrt{\frac{1}{3}} \approx \frac{77}{128}$

$= \frac{1}{\sqrt{3}} \approx \frac{77}{128}$

$= \sqrt{3} \approx \frac{128}{77}$