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3.8 Modelling with Series

① Given: $a=6$ $d=2$

a) $u_n = a+(n-1)d \Rightarrow n=5 \Rightarrow u_n = 6+(4)(2) = 8+6 = 14\text{ km}$

b) $u_n = 6+(n-1)2 = 6-2+2n = 2n+4$

c) $u_n = 42 \Rightarrow 42 = 2n+4 \Rightarrow 38 = 2n \Rightarrow n = 19\text{ weeks}$

d) $n=20;$ $\Rightarrow u_n = 2(20)+4 = 44\text{ km}$

② Given: $a = 28000$ $r = 1+0.025 = 1.025$

a) $n=2 \Rightarrow u_n = ar^{n-1} = 28000(1.025)^{2-1} = 28700$

b) $u_n = 28000 \times (1.025)^{n-1}$

c) $n=10 \Rightarrow 28000 \times (1.025)^9 = 34968$ (nearest)

d) It is unlikely that the adult population will increase by exactly the same percentage each year.

③ Given: $n=52$ $a=10p$ $d=5p$

a) $u_n = a+(n-1)d = 10+(51)5 = 265p = £2.65$

b) $S_n = \frac{n}{2}(a+L) = \frac{52}{2}(10+265) = 7150p = £71.5$

④ Given: $S_n = £6000$ $n=24$

Option 1: $a = £120$ $L = £396$ Option 2: $a = £110$ $r = 1.07$

Option 1 = $S_n = \frac{24}{2}(120+396) = £6192$

Option 1 is cheaper

Option 2 = $S_n = \frac{110(1-1.07^{24})}{1-1.07} = £6399.43$

difference = $£6399.43 - £6192 = £207.43$

⑤ Given: $a = £15000$ $r = 0.75$

a) $£15000 \times (0.75)^3 = £6328.125 = £6328$

b) $15000(0.75)^{n-1} < 1000$ $\quad n-1 < \log\left[\frac{1}{15}\right] \times \frac{1}{\log(0.75)}$

$\Rightarrow n-1 < 9.4$ $\quad n < 10.4$ $\quad \boxed{n=10}$

⑥ Given: $a = £250 \quad r = 1.05 \quad n = 4$

a) $u_n = ar^{n-1} = 250(1.05)^3 = £289.41$ (2dp)

b) $S_{10} = \dfrac{250(1-1.05^{10})}{1-1.05} = £3144.47$ (2dp)

c)

$a = £225$

$a_2 = £236$

$a_3 = £248.$

$a_4 = £260.47$

$a_5 = £273.49$

$a_6 = £287.$

$a_7 = £301.52$

$a_8 = £316.78$

$a_9 = £334.$

$a_{10} = £352.83$

$\Rightarrow a_1 + a_2 + a_3 + a_4 + a_5 \cdots + a_{10} = £2836$

⑦ Given: $a = 2 \quad r = \frac{3}{4}$

a) $2\left(\frac{3}{4}\right)^{n-1} < 0.1 \Rightarrow \left(\frac{3}{4}\right)^{n-1} < 0.05 \Rightarrow n-1 < \log(0.05) \times \dfrac{1}{\log\left(\frac{3}{4}\right)} \Rightarrow n-1 < 10.4$

$n < 11.4 \quad \boxed{n = 11}$

b) $S_{10} \Rightarrow \dfrac{2(1-r^{10})}{1-r} = 7.5495 \Rightarrow 2(7.5495) - 2 = 13.10\text{m}$

c) The ball is unlikely to continue to bounce to a constant height of $\left(\frac{3}{4}\right)$ for a long period of time due to energy losses.