Author: <u>Blinzy Fernandes</u>

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Problem Solving Set A

Bronze :

a] $(1+10x)^{1/2}$

$$= 1 + \left(\frac{1}{2}\right)(10x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(10x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(10x)^3$$

$$= 1 + 5x - \frac{25}{2}x^2 + \frac{125}{2}x^3$$

So $p = -\frac{25}{2}$, $q = \frac{125}{2}$

b] $1 + 5x - \frac{25}{2}x^2 + \frac{125}{2}x^2$

as $x < \frac{1}{10} \implies$ lets use $x = \frac{1}{20} \approx 0.05$

$\Rightarrow 1 + 5(0.05) - \frac{25}{2}(0.05)^2 + \frac{125}{2}(0.05)^3$

$\Rightarrow 1.2265625$

c] Actual value $= (1.5)^{\frac{1}{2}} = 1.224744871$

Estimated value $= 1.2265625$

$$\%\ \text{error} = \frac{A-E}{A} \times 100$$

$$= \frac{1.224744871 - 1.2265625}{1.224744871} \times 100$$

$$= \underline{\underline{0.15\%}}$$

Problem Solving Set A

Silver:

a] $\sqrt{1-6x} \Rightarrow (1-6x)^{1/2}$

$$= 1 + \left(\frac{1}{2}\right)(-6x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(-6x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(6x)^3$$

$$= 1 - 3x - \frac{9}{2}x^2 - \frac{27}{2}x^3$$

b] Substitute $x = \frac{1}{9}$

$$\sqrt{1-6\left(\frac{1}{9}\right)} = 1 - 3\left(\frac{1}{9}\right) - \frac{9}{2}\left(\frac{1}{9}\right)^2 - \frac{27}{2}\left(\frac{1}{9}\right)^3$$

$$\sqrt{\frac{1}{3}} = \frac{16}{27}$$

$$\frac{1}{\sqrt{3}} = \frac{16}{27}$$

$$\sqrt{3} = \frac{27}{16}$$

c] $1 - 6x < 0$

$6x < 1$

$|x| < \frac{1}{6}$

Since, $\frac{1}{9} < \frac{1}{6}$.

Hence the expansion is valid.

Problem Solving : Set A

Gold

a $\sqrt{\dfrac{1+3x}{1-x}} \longrightarrow \dfrac{(1+3x)^{1/2}}{(1-x)^{1/2}}$

$\Rightarrow (1+3x)^{1/2}(1-x)^{-1/2}$

$(1+3x)^{1/2}$

$$= 1 + \left(\frac{1}{2}\right)(3x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)(3x)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)(3x)^3}{3!}$$

$$= 1 + \cancel{\frac{3x}{3}}\ \frac{3x}{2} - \frac{9}{8}x^2 + \frac{27}{16}x^3$$

$(1-x)^{-1/2}$

$$= 1 + \left(\frac{-1}{2}\right)(-x) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)(-x)^2}{2!} + \frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)\left(\frac{-1}{2}-2\right)(-x)^3}{3!}$$

$$= 1 + \frac{x}{2} + \frac{3}{8}x^2 + \frac{5}{16}x^3$$

$$= \left(1 + \frac{3x}{2} - \frac{9}{8}x^2 + \frac{27}{16}x^3\right)\left(1 + \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{16}\right)$$

Problem Solving Set A

Gold (Cont.)

a) $= 1 + \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{16} + \frac{3x}{2} + \frac{3x^2}{4}$

$+ \frac{9x^3}{16} - \frac{9x^2}{8} - \frac{9x^3}{16} + \frac{27x^3}{16}$

$= 1 + 2x + 2x^3$

Hence, proved.

b) $1 + 3x < 0$

$3x < -1$

$|x| < \frac{1}{3}$

Since, the expansion is valid for $x < \frac{1}{3}$

And $x = \frac{1}{2}$ is $> \frac{1}{3}$

$\left(\frac{1}{2} > \frac{1}{3}\right)$, therefore the expansion is not valid and there is a error in the students working.

c) $\sqrt{\frac{1+3x}{1-x}} = 1 + 2x + 2x^3$

$\sqrt{5} \approx 2.236067977$

$x = \frac{1}{16} \Rightarrow \frac{\sqrt{285}}{15} \Rightarrow \sqrt{5} \times \frac{\sqrt{57}}{15}$

$= 1 + 2\left(\frac{1}{16}\right) + 2\left(\frac{1}{16}\right)^3$

$x = \frac{1}{16} \Rightarrow \sqrt{5} \approx 2.236118\ldots$

Problem Solving : Set B

Bronze

a] $\frac{1}{2-3x} = (2-3x)^{-1} \Rightarrow 2^{-1}\left(1-\frac{3x}{2}\right)^{-1}$

$= 2^{-1}\left(1 + (-1)\left(\frac{-3x}{2}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{-3x}{2}\right)^2\right)$

$= \frac{1}{2}\left(1 + \frac{3x}{2} + \frac{9}{4}x^2\right)$

$= \frac{1}{2} + \frac{3x}{4} + \frac{9x^2}{8}$ Hence, proved

$2-3x < 0$

$-3x < -2$

$|x| < \frac{2}{3}$

b] $\frac{1}{(1-x)^2} \Rightarrow (1-x)^{-2}$

$\Rightarrow \left(1 + (-2)(-x) + \frac{(-2)(-2-1)}{2!}(-x)^2\right)$

$\Rightarrow 1 + 2x + 3x^2$

c] $\frac{3x^2-4}{(2-3x)(1-x)^2} = \frac{A}{(2-3x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$

$= A(1-x)^2 + B(2-3x)(1-x) + C(2-3x)$

$= A(1-2x+x^2) + B(2-2x-3x+3x^2) + C(2-3x)$

$= A(1-2x+x^2) + B(2-5x+3x^2) + C(2-3x)$

Problem Solving : Set B

<u>Bronze</u> Cont.

c] $x^2 = A + 3B + 0 = 3$
$x = -2A - 5B - 3C = 0$
$c = A + 2B + 2C = -4$

$A = -24, \quad B = 9, \quad C = 1$

d] $\frac{-24}{(2-3x)} + \frac{9}{(1-x)} + \frac{1}{(1-x)^2}$

$\frac{9}{1-x} \Rightarrow 9(1-x)^{-1}$

$\Rightarrow 9\left(1 + (-1)(-x) + \frac{(-1)(-1-1)}{2!}(-x)^2\right)$

$\Rightarrow 9(1 + x + x^2)$

$\Rightarrow 9 + 9x + 9x^2$

$\Rightarrow -24(2-3x)^{-1} + 9(1-x)^{-1} + 1(1-x)^{-2}$

$\Rightarrow -24\left(\frac{1}{2} + \frac{3x}{4} + \frac{9x^2}{8}\right) + 9(1 + x + x^2) + 1(1 + 2x + 3x^2)$

$\Rightarrow -12 - 18x - 27x^2 + 9 + 9x + 9x^2 + 1 + 2x + 3x^2$

$\Rightarrow -2 - 7x - 15x^2$

e] $2 - 3x < 0$
$-3x < -2$
$|x| < \frac{2}{3}$

Problem Solving : Set B

Silver

a] $f(x) = \frac{8x^2 + 12x + 4}{x^2 + x - 6} \Rightarrow \frac{8x^2 + 12x + 4}{(x-2)(x+3)}$

$$\Rightarrow \frac{8x^2 + 12x + 4}{(x-2)(x+3)} = A + \frac{B}{x-2} + \frac{C}{x+3}$$

$$= A(x-2)(x+3) + B(x+3) + C(x-2)$$
$$= A(x^2 + x - 6) + B(x+3) + C(x-2)$$

$x^2 \Rightarrow \quad A + 0 + 0 = 8$
$x \Rightarrow \quad A + B + C = 12$
$c \Rightarrow \quad -6A + 3B - 2C = 4$

$A = 8 \qquad B = 12 \qquad C = -8$

$$\Rightarrow 8 + \frac{12}{x-2} - \frac{8}{x+3}$$

b] $\frac{12}{x-2} \Rightarrow 12(x-2)^{-1} \Rightarrow 12(-2+x)^{-1}$

$$\Rightarrow 12 \times (-2)^{-1}\left(1 - \frac{x}{2}\right)^{-1}$$

$$\Rightarrow -6\left(1 - \frac{x}{2}\right)^{-1}$$

$\frac{-8}{x+3} \Rightarrow -8(x+3)^{-1} \Rightarrow -8(3+x)^{-1}$

$$\Rightarrow -8 \times (3)^{-1}\left(1 + \frac{x}{3}\right)^{-1}$$

$$\Rightarrow \frac{-8}{3}\left(1 + \frac{x}{3}\right)^{-1}$$

Problem Solving : Set B

Silver : Cont.

b] $\Rightarrow 8 - 6\left(1-\frac{x}{2}\right)^{-1} - \frac{8}{3}\left(1+\frac{x}{3}\right)^{-1}$

$\rightarrow \quad 8 - 6\left(1+(-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-1-1)}{2!}\left(-\frac{x}{2}\right)^2\right.$

$\left. - \frac{8}{3}\left(1+(-1)\left(\frac{x}{3}\right) + (-1)(-1-1)\left(\frac{x}{3}\right)^2\right)\right)$

$\rightarrow \quad 8 - 6\left(1+\frac{x}{2}+\frac{x^2}{4}\right) - \frac{8}{3}\left(1-\frac{x}{3}+\frac{x^2}{9}\right)$

$\Rightarrow \quad 8 - 6 - 3x - \frac{3x^2}{2} - \frac{8}{3} + \frac{8x}{9} - \frac{8x^2}{27}$

$f(x) = -\frac{2}{3} - \frac{19}{9}x - \frac{97}{54}x^2$

c] $x - 2 < 0$

$x < 2$

$|x| < 2$

Gold

$2x^2+5x+3 \Rightarrow (2x+3)(x+1)$

$\frac{1}{2x^2+5x+3} \Rightarrow \frac{1}{(2x+3)(x+1)} = \frac{A}{2x+3} + \frac{B}{x+1}$

$A(x+1) + B(2x+3)$

$x \Rightarrow A + 2B = 0$

$c \Rightarrow A + 3B = 1$

$A = -2, \quad B = +1$

Problem Solving : Set B

Gold : Cont.

So $\frac{1}{2x^2+5x+3} = \frac{-2}{2x+3} + \frac{1}{x+1}$

$\frac{-2}{2x+3} \Rightarrow -2(2x+3)^{-1} \Rightarrow -2(3+2x)^{-1}$

$\Longrightarrow -2 \times (3)^{-1} \times \left(1+\frac{2x}{3}\right)^{-1}$

$\Rightarrow -\frac{2}{3}\left(1+\frac{2x}{3}\right)^{-1}$

$\Rightarrow \frac{1}{x+1} = 1(x+1)^{-1} = (1+x)^{-1}$

$\Longrightarrow -\frac{2}{3}\left(1+\frac{2x}{3}\right)^{-1} + (1+x)^{-1}$

$\Longrightarrow -\frac{2}{3}\left(1-\frac{2x}{3}+\frac{4x^2}{9}\right) + (1-x+x^2)$

$\Longrightarrow -\frac{2}{3} + \frac{4x}{9} - \frac{8x^2}{27} + 1 - x + x^2$

$\Longrightarrow \frac{1}{3} - \frac{5x}{9} + \frac{19x^2}{27}$

Expanding the binomial expansion $\sqrt{1+3x}$

$\Longrightarrow (1+3x)^{1/2} \Longrightarrow 1 + \left(\frac{1}{2}\right)(3x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(3x)^2$

$\Longrightarrow 1 + \frac{3x}{2} - \frac{9x^2}{8}$

Problem Solving : Set B

Gold : Cont.

So,

$$\frac{\sqrt{1+3x}}{2x^2+5x+3} \Rightarrow \left(1+\frac{3x}{2}-\frac{9x^2}{8}\right)\left(\frac{1}{3} \mp \frac{5x}{9}+\frac{19x^2}{27}\right)$$

$$\Rightarrow \frac{1}{3}-\frac{5x}{9}+\frac{19x^2}{27}+\frac{3x}{6}-\frac{15x^2}{18}-\frac{9x^2}{27}$$

$$\Rightarrow \frac{1}{3}-\frac{x}{18}-\frac{109x^2}{216}$$

Hence, proved.