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3·1: Arithmetic sequences

(1) a) $4n+3$

i) $a_1 = 4(1)+3 = 7$
$a_2 = 4(2)+3 = 11$
$a_3 = 4(3)+3 = 15$
$a_4 = 4(4)+3 = 19$

ii) $a = 3 \quad d = 7$

b) $7-3n$

i) $a_1 = 7-3(1) = 4$
$a_2 = 7-3(2) = 1$
$a_3 = 7-3(3) = -2$
$a_4 = 7-3(4) = -5$

ii) $a = 4 \quad d = 3$

c) $8+\frac{1}{2}n$

i) $a_1 = 8+\frac{1}{2}(1) = \frac{17}{2}$
$a_2 = 8+\frac{1}{2}(2) = 8$
$a_3 = 8+\frac{1}{2}(3) = \frac{19}{2}$
$a_4 = 8+\frac{1}{2}(4) = 10$

ii) $a = \frac{17}{2} \quad d = \frac{1}{2}$

d) $2n-5$

i) $a_1 = 2(1)-5 = -3$
$a_2 = 2(2)-5 = -1$
$a_3 = 2(3)-5 = 1$
$a_4 = 2(4)-5 = 3$

ii) $a = -3 \quad d = 2$

(2) a) $2, 5, 8, \ldots, 95, 98$

Given: $a = 2 \quad d = 3 \quad u_n = 98 \quad u_n = a+(n-1)d$

$\Rightarrow 98 = 2+(n-1)3 \Rightarrow 98 = 3n-1 \Rightarrow 99 = 3n$ **$n = 33$**

b) $75, 73, 71, \ldots, -3, -5$

Given: $a = 75 \quad d = -2 \quad u_n = -5 \quad u_n = a+(n-1)d$

$\Rightarrow -5 = 75+(n-1)-2 \Rightarrow -5 = 77-2n \Rightarrow 182 = 2n$ **$n = 41$**

c) $-3, 1, 5, \ldots, 109, 113$

Given: $a = -3$ $d = 4$ $u_n = 113$ $u_n = a + (n-1)d$

$\Rightarrow 113 = -3 + (n-1)4$ $\Rightarrow 113 = -7 + 4n$ $\Rightarrow 120 = 4n$ $\boxed{n = 30}$

d) $-4k, -2k, 0, \ldots, 98k, 100k$

Given: $a = -4k$ $d = 2k$ $u_n = 100k$ $u_n = a + (n-1)d$

$\Rightarrow 100k = -4k + (n-1)2k$ $\Rightarrow 100k = -4k + 2nk - 2k$ $\Rightarrow 106k = 2nk$

$\boxed{n = 53}$

(2) a) $2, 6, 10, 14 \ldots$

Given: $a = 2$ $d = 4$ $u_n = a + (n-1)d$

$\Rightarrow u_n = 2 + 4(n-1) = 2 - 4 + 4n = 4n - 2$

$a_{10} = 4(10) - 2 = 38$

b) $5, 3, 1, -1, \ldots$

Given: $a = 5$ $d = -2$ $u_n = a + (n-1)d$

$\Rightarrow u_n = 5 - 2(n-1) = 5 + 2 - 2n = 7 - 2n$

$a_{10} = 7 - 2(10) = 7 - 20 = -13$

c) $-2, 1, 4, 7, \ldots$

Given: $a = -2$ $d = 3$ $u_n = a + (n-1)d$

$\Rightarrow u_n = -2 + 3(n-1) = -2 - 3 + 3n = 3n - 5$

$a_{10} = 3(10) - 5 = 25$

d) $8y, 5y, 2y, -y, \ldots$

Given: $a = 8y$ $d = -3y$ $u_n = a + (n-1)d$

$\Rightarrow u_n = 8y - 3y(n-1) = 8y + 3y - 3yn = 11y - 3yn$

$a_{10} = 11y - 3y(10) = -19y$

④ Given: $a_4 = 17$ $a_{10} = 47$ $u_n = a + (n-1)d$

$a_4 = a + 3d = 17$ — ① $\quad a_{10} = a + 9d = 47$ — ②

$a + 9d = 47$

$a + 3d = 17$

$6d = 30$ $\quad d = 5$

$a + 15 = 17$ (sub $d = 5$ in eq ①)

$a = 2$

⑤ Given: $a = 20$ $\quad d = -\frac{1}{2}$

a) $a + (n-1)d = u_n$ $\quad a_{25} = 20 + 24\left(-\frac{1}{2}\right) = 8$

b) $r^{th} = 0$ $\quad 0 = 20 + (r-1)-\frac{1}{2}$

$\Rightarrow \quad 20 - \frac{1}{2}r + \frac{1}{2} = 0 \quad -\frac{1}{2}r = -20.5 \quad r = 41$

⑥ Given: $3n - 4 = u_n$

a) $a_1 = 3(1) - 4 = -1 \quad a_2 = 3(2) - 4 = 2 \quad a_3 = 3(3) - 4 = 5$

b) $d = 3 \quad (5 - 2 = 3)(a_3 - a_2)$

⑦ Given: $a_4 = -2$ $\quad a_9 = -22$ $\quad u_n = a + (n-1)d$

a) $-2 = a + 3d$ — ① $\quad -22 = a + 8d$ — ②

$a + 8d = -22$

$a + 3d = -2$

$5d = -20$ $\quad d = -4$ → sub in eq ①

$-2 = a - 12$ $\quad a = 10$

b) $u_n = -54 \quad a = 10 \quad d = -4$

$\Rightarrow -54 = 10 + (n-1)-4$ $\boxed{n = 17}$

$\Rightarrow -54 = 10 - 4n + 4$

$-68 = -4n$

(8) Given :- a, b, c

a) $d_1 = b - a \quad d_2 = c - b \quad d_1 = d_2$

$b - a = c - b$ $\boxed{b = \frac{c+a}{2}}$

$b + b = c + a$

$2b = c + a$

b) Given: $15, p^2, -p$

$d_1 = p^2 - 15 \quad d_2 = -p - p^2$ $\boxed{d_1 = d_2}$

$p^2 - 15 = -p - p^2 \Rightarrow 2p^2 + p - 15 = 0$

$(2p - 5)(p + 3) = 0$

$\boxed{p = \frac{5}{2}}$ or $\boxed{p = 3}$

(9) Given: $a = k \quad a_2 = 4k - 3 \quad a_3 = 3k + 10$

a) $d_1 = a_2 - a = 4k - 3 - k = 3k - 3$

$d_2 = a_2 - a_3 = 3k + 10 - 4k + 3 = 13 - k$

$d_1 = d_2 \Rightarrow 3k - 3 = 13 - k$

$\Rightarrow 4k = 16$ $\boxed{k = 4}$

b) $a = 4 \quad a_2 = 13 \quad a_3 = 22 \quad d = 9$

$u_n = a + (n-1)d$ $\quad u_{50} = 4 + 49(9) = 445$