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5.5 : Small angle approximations

$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta = 1 - \frac{\theta^2}{2}$

① a) $\frac{\sin 3\theta}{\theta} = \frac{3\cancel{\theta}}{\cancel{\theta}} = 1$ b) $\frac{\tan 4\theta}{\sin 2\theta} = \frac{4\cancel{\theta}}{2\cancel{\theta}} = 2$

c) $\frac{5\tan\theta - \theta}{\sin 4\theta} = \frac{5\theta - \theta}{4\theta} = \frac{4\theta}{4\theta} = 1$

② a) $\frac{1-\cos 2\theta}{\theta\sin\theta} = \frac{1 - \left(1 - \frac{(2\theta)^2}{2}\right)}{\theta^2} \Rightarrow \frac{1 - \left(\frac{2 - 4\theta^2}{2}\right)}{\theta^2} = \frac{\frac{\cancel{2} - \cancel{2} + \cancel{4}\theta^2}{\cancel{2}}}{\theta^2}$

$\Rightarrow \frac{+2\cancel{\theta^2}}{\cancel{\theta^2}} = 2$

b) $\frac{\cos 6\theta - 1}{\sin 3\theta \tan\theta} = \frac{\cancel{1} - \frac{(6\theta)^2}{2} \cancel{-1}}{\theta \times 3\theta} = \frac{-36\theta^2}{2} \times \frac{1}{3\theta^2} = \frac{-36\theta^2}{6\theta^2} = -6$

c) $\frac{\theta\tan\theta}{1-\cos\theta} = \frac{\theta \times \theta}{\cancel{1} - \cancel{1} + \frac{\theta^2}{2}} = \frac{2\theta^2}{\theta^2} = 2$

③ a) $\frac{2\theta - 3\sin\theta}{\theta\tan 4\theta} = -\frac{1}{4\theta} \Rightarrow \frac{2\theta - 3\theta}{\theta \times 4\theta} = \frac{-\cancel{\theta}}{4\cancel{\theta^2}\theta} = \frac{-1}{4\theta}$

b) $\frac{\cos\theta - 1}{\sin\theta} = -\frac{1}{2}\theta \Rightarrow \frac{\cancel{1} - \frac{\theta^2}{2} \cancel{-1}}{\theta} = -\frac{\cancel{\theta^2}^{\theta}}{2} \times \frac{1}{\cancel{\theta}} = \frac{-1}{2}\theta$

c) $\frac{\sin\theta + \tan\theta}{1 - \cos 2\theta} = \frac{\theta + \theta}{1 - \left(1 - \frac{(2\theta)^2}{2}\right)} = \frac{2\theta}{\cancel{1} - \cancel{1} + \frac{\cancel{4}^{2}\theta^2}{2}} = \frac{\cancel{2}\theta}{\cancel{2}\theta^2} = \frac{1}{\theta}$

④ $\dfrac{1-\cos 2\theta}{3\theta \sin 2\theta} = \dfrac{1-\left(1-\frac{(2\theta)^2}{2}\right)}{3\theta \times 2\theta} = \dfrac{\cancel{1}-\cancel{1}+\frac{\cancel{4}^{2}\theta^2}{\cancel{2}}}{6\theta^2} = \dfrac{2\cancel{\theta^2}}{6\cancel{\theta^2}} = \dfrac{1}{3}$

⑤ $\dfrac{3\tan\theta - 4\cos\theta + 5}{\sin\theta + 1} \simeq 2\theta+1 \Rightarrow \dfrac{3\theta - 4\left(1-\frac{\theta^2}{2}\right)+5}{\theta+1}$

$\Rightarrow \dfrac{3\theta - 4 + \frac{\cancel{4}^{2}\theta^2}{\cancel{2}}+5}{\theta+1} \Rightarrow \dfrac{2\theta^2+3\theta+1}{\theta+1} \Rightarrow \dfrac{(2\theta+1)\cancel{(\theta+1)}}{\cancel{\theta+1}}$

$\Rightarrow 2\theta+1$

⑥ $5\cos^2\theta + 6\cos\theta + 1 \simeq 12 - 8\theta^2$

$\Rightarrow 5\left[1-\frac{\theta^2}{2}\right]^2 + 6\left[1-\frac{\theta^2}{2}\right]+1 \Rightarrow 5\left(1+\frac{\theta^4}{4}-\theta^2\right)+6-\frac{6\theta^2}{2}+1$

$\Rightarrow 5+\frac{5\theta^4}{\cancel{4}}-5\theta^2+6-3\theta^2+1 \Rightarrow 12-8\theta^2$

b) Josh is incorrect. He used degrees and not radians.

$\theta = \frac{10\pi}{180} \Rightarrow 12-8\left(\frac{10\pi}{180}\right)^2 = 11.76$. So $\boxed{\theta = 10^\circ}$ gives a good approximation.