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3.6 Sigma notation

(1) a) Given: $\sum_{r=1}^{6} (4r-3)$

i) $r=1 = 1$ $\quad r=4 = 13$

$r=2 = 5$ $\quad r=5 = 17$

$r=3 = 9$ $\quad r=6 = 21$

$\Rightarrow 1+5+9+13+17+21$

ii) $a=5 \quad d=4 \quad n=6$

$S_n = 3(1+21) = 3(22) = 66$

b) Given: $\sum_{r=1}^{5} 2r^3$

i) $r=1 = 2$ $\quad r=4 = 128$

$r=2 = 16$ $\quad r=5 = 250$

$r=3 = 54$

$\Rightarrow 2+16+54+128+250$

ii) it is neither arithmetic or geometric.

$S_n = 2+16+54+128+250 = 450$

c) Given: $\sum_{r=0}^{4} \cos(90r^\circ)$

i) $r=0 = 1$ $\quad r=3 = 0$

$r=1 = 0$ $\quad r=4 = 1$

$r=2 = -1$

$\Rightarrow 1+0-1+0+1$

ii) $\cancel{1}+0\cancel{-1}+0+1 = 1$

d) Given: $\sum_{r=3}^{7} 3\left(-\frac{1}{2}\right)^r$

i) $r=3 = -\frac{3}{8}$ $\quad r=6 = 3/64$

$r=4 = \frac{3}{16}$ $\quad r=7 = -3/128$

$r=5 = -\frac{3}{32}$

$\Rightarrow -\frac{3}{8} + \frac{3}{16} - \frac{3}{32} + \frac{3}{64} - \frac{3}{128}$

ii) $a = -\frac{3}{8}$ $\quad n=5$ $\quad r = -1/2$

$$S_n = \frac{-\frac{3}{8}\left(1-\left(-\frac{1}{2}\right)^5\right)}{1+\frac{1}{2}} = -\frac{33}{128}$$

② Given: $u_n = 5n-1$

a) $a = 5(1)-1 = 4$ $\quad a_1 = 4$ $\quad a_2 = 5(2)-1 = 9$

$d = 5$

b) $\sum_{r=1}^{12} (5r-1)$ $\quad a = 4$ $\quad l = 59$ $\quad S_n = \frac{n}{2}(a+l)$

$d = 5$

$n = 12$ $\quad \Rightarrow \frac{12}{2}(4+59) = 378$

③ Given: $u_n = 3 \times 2^{n-1}$

a) $a = 3 \times 2^0 = 3$ $\quad a_2 = 3 \times 2^1 = 6$ $\quad r = \frac{6}{3} = 2$

b) $a = 3$ $\quad r = 2$ $\quad n = 20$ $\quad S_n = a\left(\frac{1-r^n}{1-r}\right)$

$$\sum_{r=1}^{20} 3 \times 2^{r-1} = \frac{3(1-2^{20})}{1-2} = 3145725$$

4 a) Given: $\sum_{r=1}^{10} (3r+2) \Rightarrow r=1 = a = 5 \quad r=3 = 11 = a_3$

$r=2 = 8 = a_2$

$= 5+8+11 \dots 32$

$a=5 \quad d=3 \quad l=32 \quad n=10 \quad S_n = 5(5+32) = \boxed{185}$

b) Given: $\sum_{r=1}^{20} 3 \Rightarrow 3(20) = \boxed{60}$

c) Given: $\sum_{r=1}^{\infty} 5\left(\frac{2}{3}\right)^r \Rightarrow r=1 = 10/3 \quad a = 10/3$

$r=2 = 20/9 \quad r = 2/3$

$r=3 = 40/27$

$$S_\infty = \frac{a}{1-r} = \frac{\frac{10}{3}}{1-\frac{2}{3}} = \frac{10/\cancel{3}}{1/\cancel{3}} = \boxed{10}$$

5 a) Given: $\sum_{r=1}^{30} (5-2r) = \sum_{r=1}^{7} (5-2r) + \sum_{r=8}^{30} (5-2r)$

so, $\sum_{r=8}^{30} (5-2r) = \sum_{r=1}^{30} (5-2r) - \sum_{r=1}^{7} (5-2r)$

b) $\sum_{r=8}^{30} (5-2r) \Rightarrow a = 5-2(8) = -11 \quad l = -55 \quad n = 23$

$a_2 = 5-2(9) = -13$

$\Rightarrow S_n = \frac{23}{2}(-11-55) = \boxed{-759}$

6 Given: $\sum_{r=10}^{15} 4 \times 3^{r-1} \Rightarrow a = 4 \times 3^{10-1} = 78732$

$n=6 \quad r=3 \quad \Rightarrow a_2 = 4 \times 3^{10} = 236196$

$\Rightarrow$ geometric series $= S_n = \frac{a(1-r^n)}{1-r} = \frac{78732(1-3^6)}{1-3} = \boxed{28658448}$

⑦ Given: $\sum_{r=1}^{10} 10\times 3^r \Rightarrow a = 10\times 3^1 = 30 \quad r=3 \quad S_n = \frac{a(1-r^n)}{1-r}$

$a_2 = 10\times 3^2 = 90 \quad n=10$

$\Rightarrow \frac{30(1-3^{10})}{1-3} = 885720$

⑧ Given: $\sum_{r=1}^{k} (3r+6) = 750 \Rightarrow a = 3(1)+6 = 9 \quad d=3 \quad S_n = \frac{n}{2}\left(2a+(n-1)d\right)$

$a_2 = 3(2)+6 = 12 \quad n=k$

a) $\Rightarrow 750 = \frac{k}{2}\left(2(9)+(k-1)3\right) \Rightarrow 750 = \frac{k}{2}(18+3k-3)$

$\Rightarrow 1500 = k(15+3k) \Rightarrow 3k^2+15k-1500=0 \Rightarrow (k-20)(3k+75)=0$

b) $k=20$ or $\cancel{k=-25}$ $\quad n>0,\ k>0$

⑨ Given: $\sum_{r=1}^{k} 4\times 2^r = 262136 \quad a=8 \quad r=2 \quad n=k \quad S_n = \frac{a(1-r^n)}{1-r}$

$a_2 = 16 \quad S_n = 262136$

a) $262136 = \frac{8(1-2^k)}{1-2} \Rightarrow -262136 = 8(1-2^k)$

$\Rightarrow -32767 = 1-2^k \Rightarrow \cancel{-}32768 = \cancel{-}2^k \Rightarrow \ln(32768) = k(\ln 2)$

$k = \frac{\ln(32768)}{\ln 2}$

b) $k=15 \Rightarrow \sum_{r=k+1=16}^{20} 4\times 2^r \quad a = 262144 \quad r=2 \quad n=5$

$a_2 = 524288$

$S_{20} = \frac{262144(1-2^5)}{1-2} = 8126464$

⑩ Given: $1+4x+16x^2+\dots$ (convergent) $\rightarrow |r|<1$

a) $a=1 \quad ar = 4x$

$r = 4x \quad |4x|<1 \quad |x|<\frac{1}{4}$

b) $\sum_{r=1}^{\infty} (4x)^{r-1} = 5 \quad a = (4x)^0 = 1 \quad r = 4x \quad S_\infty = \frac{a}{1-r} = \frac{1}{1-4x} = 5$

$\Rightarrow \quad 1 = 5(1-4x) \quad \Rightarrow \quad 1 = 5 - 20x \quad \Rightarrow \quad \cancel{-}4 = \cancel{-}20x \quad \boxed{x = \frac{1}{5}}$

(11) Given: $\sum_{r=1}^{10} (4 + 3r + 2^{r-1}) = 1228$

$$\sum_{r=1}^{10}(4) + \sum_{r=1}^{10}(3r) + \sum_{r=1}^{10}(2^{r-1}) = 1228$$

$\sum_{r=1}^{10}(4) = 10 \times 4 = 40$; $\sum_{r=1}^{10}(3r) = 3 \times \left[\frac{10 \times 11}{2}\right] = 165$; $\sum_{r=1}^{10}(2^{r-1}) = \frac{1(2^{10}-1)}{2-1} = 1023$

$\frac{n(n+1)}{2}$

$$40 + 165 + 1023 = 1228$$