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4.3 Using Partial Fractions

1a] $\frac{4x+1}{(1+x)(2-x)} = \frac{A}{1+x} + \frac{B}{2-x}$

$A(2-x) + B(1+x)$

Constant $\longrightarrow 2A + B = 1$

$x \longrightarrow -A + B = 4$

$\therefore A = -1, \quad B = 3$

$\Rightarrow \frac{-1}{1+x} + \frac{3}{2-x}$

b] $-1(1+x)^{-1} + 3(2-x)^{-1}$

$-1(1+x)^{-1} + 3(2)^{-1}\left(1 - \frac{x}{2}\right)^{-1}$

$-1(1+x)^{-1} + \frac{3}{2}\left(1 - \frac{x}{2}\right)^{-1}$

$\Rightarrow -1\left(1 + (-1)(+x) + \frac{(-1)(-1-1)}{2!}(+x)^2\right)$

$= -1(1 + x + x^2)$

$= -1 + x - x^2$ —— ①

$\Rightarrow \frac{3}{2}\left(1 + (-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-1-1)}{2!}\left(-\frac{x}{2}\right)^2\right)$

$= \frac{3}{2}\left(1 + \frac{x}{2} + \frac{x^2}{4}\right)$

$= \frac{3}{2} + \frac{3}{4}x + \frac{3}{8}x^2$ —— ②

Equate ① and ② together.

4.3 Using Partial Fractions

1b] Cont.

$$-1 + x - x^2 + \frac{3}{2} + \frac{3}{4}x + \frac{3}{8}x^2$$

$$\Rightarrow \frac{1}{2} + \frac{7}{4}x - \frac{5}{8}x^2$$

c] $1 + x < 0$

$|x| < 1$

2a] $$\frac{3x}{(3+x)^2} = \frac{A}{3+x} + \frac{B}{(3+x)^2}$$

$$\Rightarrow \frac{A(3+x)^2}{3+x} + \frac{B(3+x)}{(3+x)^2}$$

$$\Rightarrow \frac{A(3+x) + B}{(3+x)^2}$$

Equating the numerator

$x \longrightarrow A + 0 = 3$

constant $\rightarrow 3A + B = 0$

$\therefore A = 3, \quad B = -9$

$$\Rightarrow \frac{3}{3+x} - \frac{9}{(3+x)^2}$$

b] $3(3+x)^{-1} - 9(3+x)^{-2}$

$$\Rightarrow 3 \times 3^{-1}\left(1 + \frac{x}{3}\right)^{-1} - 9 \times 3^{-2}\left(1 + \frac{x}{3}\right)^{-2}$$

4.3 Using Partial Fractions

26] Cont.

$= 1\left(1+\frac{x}{3}\right)^{-1} - 1\left(1+\frac{x}{3}\right)^{-2}$

$= 1\left(1 + (-1)\left(\frac{x}{3}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{x}{3}\right)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}\left(\frac{x}{3}\right)^3\right.$

$= 1 - \frac{x}{3} + \frac{1}{9}x^2 - \frac{1}{27}x^3$ ——— ①

$= -1\left(1 + (-2)\left(\frac{x}{3}\right) + \frac{(-2)(-2-1)}{2!}\left(\frac{x}{3}\right)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}\left(\frac{x}{3}\right)^3\right.$

$= -1\left(1 - \frac{2x}{3} + \frac{1}{3}x^2 - \frac{4}{27}x^3\right)$

$= -1 + \frac{2x}{3} - \frac{1}{3}x^2 + \frac{4}{27}x^3$ ——— ②

Equating ① and ② together

$= 1 - \frac{x}{3} + \frac{x^2}{9} - \frac{1}{27}x^3 - 1 + \frac{2x}{3} - \frac{1}{3}x^2 + \frac{4x^3}{27}$

$= \frac{1}{3}x - \frac{2}{9}x^2 + \frac{1}{9}x^3$

2c] $3 + x < 0$

$|x| < 3$

3a] $\frac{2x^2+5x-8}{(x-5)(x+4)} = A + \frac{B}{(x-5)} + \frac{C}{(x+4)}$

$= A(x-5)(x+4) + B(x+4) + C(x-5)$

$= A(x^2 - x - 20) + B(x+4) + C(x-5)$

4.3 Using Partial Fractions

3a] Cont.

Equating terms :-

$x^2 \longrightarrow A + 0 + 0 = 2$

$x \longrightarrow -A + B + C = 5$

$c \longrightarrow -20A + 4B - 5C = -8$

So $A = 2$, $B = \frac{67}{9}$, $C = -\frac{4}{9}$

$$= 2 + \frac{67}{9(x-5)} - \frac{4}{9(x+4)}$$

b] $2 + \frac{67}{9}(x-5)^{-1} - \frac{4}{9}(x+4)^{-1}$

$= 2 + \frac{67}{9}(-5+x)^{-1} - \frac{4}{9}(4+x)^{-1}$

$= 2 + \frac{67}{9} \times (5)^{-1}\left(1 - \frac{x}{5}\right)^{-1} - \frac{4}{9} \times (4)^{-1}\left(1 + \frac{x}{4}\right)^{-1}$

$= 2 - \frac{67}{45}\left(1 - \frac{x}{5}\right)^{-1} - \frac{1}{9}\left(1 + \frac{x}{4}\right)^{-1}$

$\Rightarrow -\frac{67}{45}\left(1 + (-1)\left(\frac{-x}{5}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{-x}{5}\right)^2\right)$

$-\frac{67}{45}\left(1 + \frac{x}{5} + \frac{x^2}{25}\right)$

$-\frac{67}{45} - \frac{67x}{225} - \frac{67}{1125}x^2$

$\Rightarrow -\frac{1}{9}\left(1 + (-1)\left(\frac{x}{4}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{x}{4}\right)^2\right)$

4.3 Using Partial Fractions

3b] Cont.

$$-\frac{1}{9}\left(1-\frac{x}{4}+\frac{1}{16}x^2\right)$$

$$-\frac{1}{9}+\frac{1}{36}x-\frac{1}{144}x^2$$

Equate them together

$$2-\frac{67}{45}-\frac{67}{225}x-\frac{67}{1125}x^2-\frac{1}{9}+\frac{1}{36}x-\frac{1}{144}x^2$$

$$=\frac{2}{5}-\frac{27}{100}x-\frac{133}{2000}x^2$$

c] $x+4<0$

$|x|<4$

4a] $f(x)=\dfrac{8x-1}{(1-2x)(1+4x)}=\dfrac{A}{1-2x}+\dfrac{B}{1+4x}$

$=A(1+4x)+B(1-2x)$

Equating terms

$x \Longrightarrow 4A-2B=8$

$c \Longrightarrow A+B=-1$

So, $A=1$, $B=-2$

$$=\frac{1}{1-2x}-\frac{2}{1+4x}$$

b] $(1-2x)^{-1}-2(1+4x)^{-1}$

$$\Rightarrow (1)+(-1)(-2x)+\frac{(-1)(-1-1)}{2!}(-2x)^2$$

$$=1+2x+4x^2$$

4.3 Using Partial Fractions

4b] Cont.

$$-2\left(1 + (-1)(4x) + \frac{(-1)(-1-1)}{2!}(4x)^2\right)$$

$$= -2 + 8x - 32x^2$$

Equating them together

$$= 1 + 2x + 4x^2 - 2 + 8x - 32x^2$$

$$= -1 + 10x - 28x^2$$

c) $1 + 4x < 0$

$$|x| < \frac{1}{4}$$

5a] $$\frac{10x + 4}{(1+2x)^2} = \frac{A}{1+2x} + \frac{B}{(1+2x)^2}$$

$$= A(1+2x) + B$$

Equating terms

$x \Longrightarrow 2A + 0 = 10$

$c \Longrightarrow A + B = 0$

So $A = 5$, $B = -1$

$$= \frac{5}{1+2x} - \frac{1}{(1+2x)^2}$$

b] $$\frac{5}{1+2x} - \frac{1}{(1+2x)^2}$$

$$= 5(1+2x)^{-1} - 1(1+2x)^{-2}$$

$$\Longrightarrow 5\left(1 + (-1)(2x) + \frac{(-1)(-1-1)}{2!}(2x)^2\right)$$

$$= 5 - 10x + 20x^2$$

4.3 Using Partial Fractions

5b] Cont.

$$\Rightarrow -1\left(1 + (-2)(2x) + \frac{(-2)(-2-1)}{2!}(2x)^2\right)$$

$$= -1 + 4x - 12x^2$$

Equating them together

$$= 5 - 1 - 10x + 4x + 20x^2 - 12x^2$$

$$= 4 - 6x + 8x^2$$

6a] $\dfrac{3x^2 + 5x - 7}{(x+4)(x-3)} = A + \dfrac{B}{x+4} + \dfrac{C}{x-3}$

$$\Rightarrow A(x-3)(x+4) + B(x-3) + C(x+4)$$

$$= A(x^2 + x - 12) + B(x-3) + C(x+4)$$

Equating the terms

$x^2 \Rightarrow A + 0 + 0 = 3$

$x \Rightarrow A + B + C = 5$

$C \Rightarrow -12A - 3B + 4C = -7$

So, $A = 3$, $B = -3$, $C = 5$

6b] $3 - \dfrac{3}{x+4} + \dfrac{5}{x-3}$

$$= 3 - 3(x+4)^{-1} + 5(x-3)^{-1}$$

$$= 3 - 3(4+x)^{-1} + 5(-3+x)^{-1}$$

$$= 3 - 3 \times 4^{-1}\left(1 + \frac{x}{4}\right)^{-1} + 5 \times (-3)^{-1}\left(1 - \frac{x}{3}\right)^{-1}$$

$$= 3 - \frac{3}{4}\left(1 + \frac{x}{4}\right)^{-1} - \frac{5}{3}\left(1 - \frac{x}{3}\right)^{-1}$$

$$\Rightarrow -\frac{3}{4}\left(1 + (-1)\left(\frac{x}{4}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{x}{4}\right)^2\right)$$

4.3 Using Partial Fractions

6b) Cont.

$$-\frac{3}{4}\left(1 - \frac{x}{4} + \frac{x^2}{16}\right)$$

$$-\frac{3}{4} + \frac{3x}{16} - \frac{3x^2}{64}$$

$$\Longrightarrow -\frac{5}{3}\left(1 + (-1)\left(-\frac{x}{3}\right) + \frac{(-1)(-1-1)}{2!}\left(-\frac{x}{3}\right)^2\right)$$

$$= -\frac{5}{3}\left(1 + \frac{x}{3} + \frac{1}{9}x^2\right)$$

$$= -\frac{5}{3} - \frac{5}{9}x - \frac{5}{27}x^2$$

Equating them together

$$= 3 - \frac{3}{4} - \frac{5}{3} + \frac{3x}{16} - \frac{5x}{9} - \frac{3x^2}{64} - \frac{5x^2}{27}$$

$$= \frac{7}{12} - \frac{53}{144}x - \frac{401}{1728}x^2$$

c) $x - 3 < 0$

$|x| < 3$

7a) $f(x) = \dfrac{2x^2+5x+12}{(3x-1)^2(x-2)} \equiv \dfrac{A}{3x-1} + \dfrac{B}{(3x-1)^2} + \dfrac{C}{x-2}$

$= A(3x-1)(x-2) + B(x-2) + C(3x-1)^2$

$= A(3x^2-6x-x+2) + B(x-2) + C(9x^2-6x+1)$

$= A(3x^2-7x+2) + B(x-2) + C(9x^2-6x+1)$

Equating the terms.

4.3 Using Partial fractions

7a) Cont.

$$x^2 \Rightarrow 3A + 0 + 9C = 2$$
$$x \Rightarrow -7A + B - 6C = 5$$
$$c \Rightarrow 2A - 2B + C = 12$$

So, $A = \frac{-44}{15}$, $B = \frac{-25}{3}$, $C = \frac{6}{5}$

b) $\frac{-44}{15(3x-1)} - \frac{25}{3(3x-1)^2} + \frac{6}{5(x-2)}$

$= -\frac{44}{15}(3x-1)^{-1} - \frac{25}{3}(3x-1)^{-2} + \frac{6}{5}(x-2)^{-1}$

$-\frac{44}{15}(-1+3x)^{-1} - \frac{25}{3}(-1+3x)^{-2} + \frac{6}{5}(-2+x)^{-1}$

$= -\frac{44}{15}(-1)^{-1}(1-3x)^{-1} - \frac{25}{3} \times (-1)^{-2}(1-3x)^{-2} + \frac{6}{5}(-2)^{-1}\left(1-\frac{x}{2}\right)^{-1}$

$= \frac{44}{15}(1-3x)^{-1} - \frac{25}{3}(1-3x)^{-2} - \frac{3}{5}\left(1-\frac{x}{2}\right)^{-1}$

↳ $\frac{44}{15}(1-3x)^{-1}$

$= \frac{44}{15}\left((1) + (-1)(-3x) + \frac{(-1)(-1-1)(-3x)^2}{2!}\right)$

$= \frac{44}{15}\left(1 + 3x + 9x^2\right)$

$= \frac{44}{15} + \left(+\frac{44}{5}x\right) + \frac{132}{5}x^2$

$= \frac{44}{15} + \frac{44}{5}x + \frac{132x^2}{5}$

↳ $-\frac{25}{3}(1-3x)^{-2}$

4.3 Using Partial Fractions

7b] Cont.

$$= -\frac{25}{3}\left((1) + (-2)(-3x) + \frac{(-2)(-2-1)}{2!}(-3x)^2\right)$$

$$= -\frac{25}{3}\left(1 + 6x + 27x^2\right)$$

$$= -\frac{25}{3} - 50x - 225x^2$$

$$\hookrightarrow -\frac{3}{5}\left(1 - \frac{x}{2}\right)^{-1}$$

$$= -\frac{3}{5}\left(1 + (-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-1-1)}{2!}\left(-\frac{x}{2}\right)^2\right)$$

$$= -\frac{3}{5}\left(1 + \frac{x}{2} + \frac{x^2}{4}\right)$$

$$= -\frac{3}{5} - \frac{3x}{10} - \frac{3x^2}{20}$$

Equate them together

$$\Rightarrow \frac{44}{15} + \frac{44x}{5} + \frac{132}{5}x^2 - \frac{25}{3} - 50x - 225x^2$$

$$- \frac{3}{5} - \frac{3x}{10} - \frac{3x^2}{20}$$

$$\Rightarrow -6 - \frac{83x}{2} - \frac{795x^2}{4}$$

4.3 Using Partial Fractions

7c] $3x - 1 < 0$

$3x < 1$

$x < \frac{1}{3}$

$|x| < \frac{1}{3}$

The expansion of $f(x)$ is valid for $|x| < \frac{1}{3}$

As $0.5 > \frac{1}{3}$,

the expansion is not valid for $x = 0.5$