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3.4: Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

① a) Given: $2+6+18+54\ldots$ $a=2$ $r=3$ $n=10$

$S_n = \frac{2(1-3^{10})}{1-3} = 59048$

b) Given: $a=48$ $r=\frac{1}{2}$ $n=12$

$S_n = \frac{48(1-(0.5)^{12})}{1-0.5} = 96.0$ (3sf)

c) Given: $a=3$ $r=-2$ $n=8$

$S_n = \frac{3(1-(-2)^8)}{1-(-2)} = -255$

d) Given: $a=810$ $r=2/3$ $n=9$

$S_n = \frac{810\left(1-\left(\frac{2}{3}\right)^9\right)}{1-\frac{2}{3}} = 2370$ (3sf)

② a) Given: $a=5$ $r=2$ $n=?$

$u_n = ar^{n-1}$ $\quad 640 = 5(2)^{n-1} \Rightarrow 128 = 2^{n-1}$ $\quad n-1=7$ $\quad \boxed{n=8}$

$S_n = \frac{5(1-2^8)}{1-2} = 1275$

b) Given: $a=13122$ $r=1/3$ $n=?$

$2 = ar^{n-1}$ $\quad 2 = 13122\left(\frac{1}{3}\right)^{n-1}$ $\quad n-1=8$ $\quad \boxed{n=9}$

$S_n = \frac{13122\left(1-\frac{1}{3}^9\right)}{1-\frac{1}{3}} = 19682$

c) Given: $a=1024$ $r=-\frac{1}{4}$ $n=?$

$-\frac{1}{16} = 1024\left(-\frac{1}{4}\right)^{n-1} \Rightarrow n-1=7$ $\quad \boxed{n=8}$

$$S_n = \frac{1024\left(1-\left(-\frac{1}{4}\right)^8\right)}{1+\frac{1}{4}} = 819\ (3sf)$$

d) Given: $a = 128$ $r = 3/2$ $n = ?$

$1458 = 128\left(3/2\right)^{n-1}$ $n-1 = 6$ $n = 7$

$$S_n = \frac{128\left(1-\left(3/2\right)^7\right)}{1-3/2} = 4118$$

③ Given: $a = 3$ $r = 2$ $S_n > 1000\,000$

$$\Rightarrow \frac{a(r^n-1)}{r-1} > 1000\,000 \Rightarrow \frac{3(2^n-1)}{2-1} \Rightarrow 3(2^n-1) > 1000000$$

$$2^n > \frac{1000003}{3} \qquad n > \ln\left(\frac{1000003}{3}\right) \times \frac{1}{\ln(2)}$$

$\Rightarrow n > 18.3$ $\boxed{n = 19}$

④ Given: $u_4 = 24$, $ar^3 = 24$, $u_5 = 48$, $ar^4 = 48$

a) $\frac{ar^4}{ar^3} = \frac{48}{24} \Rightarrow \boxed{r = 2}$

b) $a(2)^3 = 24$ $\boxed{a = 3}$

c) $S_{20} = \frac{3\left(1-(2)^{20}\right)}{1-2} = 3145725$

⑤ Given: $a = 10 \quad r = \frac{3}{5}$

a) $ar^9 = ? \quad 10\left(\frac{3}{5}\right)^9 = 0.101$ (3dp)

b) $S_n > 20 \Rightarrow \frac{10\left(1 - \frac{3}{5}^n\right)}{1 - \frac{3}{5}} > 20 \Rightarrow 25\left(1 - \frac{3}{5}^n\right) > 20$

$\Rightarrow 1 - \left(\frac{3}{5}\right)^n > \frac{4}{5} \Rightarrow \cancel{-}\left(\frac{3}{5}\right)^n > \cancel{-}\frac{1}{5}$

$\Rightarrow \left(\frac{3}{5}\right)^n < \frac{1}{5} \qquad n \log\left(\frac{3}{5}\right) < \log\left(\frac{1}{5}\right)$

$\Rightarrow n < 3.15 = \boxed{n = 4}$

⑥ Given: $a = 20 \quad r = 4/5 \quad S_k > 50$

a) $S_k = \frac{20\left(1 - \left(\frac{4}{5}\right)^k\right)}{1 - \frac{4}{5}} = 100\left[1 - \left(\frac{4}{5}\right)^k\right] > 50$

$\Rightarrow 1 - \left(\frac{4}{5}\right)^k > \frac{1}{2} \qquad \left(\frac{4}{5}\right)^k < 0.5 \qquad k \ln\left(\frac{4}{5}\right) < \ln 0.5$

$k > \frac{\ln 0.5}{\ln 0.8}$

b) $k > 3.1 \quad \boxed{k = 4}$

(7) Given: $a = 2k - 4 \quad ar = k+2 \quad ar^2 = k-1$

a) $\dfrac{ar^2}{ar} = \dfrac{ar}{a} \Rightarrow \dfrac{k-1}{k+2} = \dfrac{k+2}{2k-4} \Rightarrow k-1(2k-4) = (k+2)^2$

$\Rightarrow 2k^2 - 4k - 2k + 4 = k^2 + 4 + 4k$

$\Rightarrow k^2 - 10k = 0 \qquad k(k-10) = 0 \quad \boxed{k = 10}$

b) $a = 16 \quad ar = 12 \quad r = 3/4$

$u_{20} = ar^{19} = 16\left(\frac{3}{4}\right)^{19} = 0.0677 \text{ (3SF)}$

c) $S_{20} = \dfrac{16\left(1 - \left(\frac{3}{4}\right)^{20}\right)}{1 - \frac{3}{4}} = 63.8$

(8) Given: $ar = 96 \qquad a + ar + ar^2 = 292.8, \; r < 1$

$a(1 + r + r^2) = 292.8$

$\Rightarrow a = \dfrac{96}{r} \Rightarrow \dfrac{96}{r} + \dfrac{96}{\cancel{r}}(\cancel{r}) + \dfrac{96}{\cancel{r}}(r^{\cancel{2}}) = 292.8$

$\Rightarrow \dfrac{96}{r} + 96 + 96r = 292.8$

$\Rightarrow 96 + 96r - 292.8 + 96r^2 = 0$

$\cancel{r = \frac{5}{4}}$ or $r = \dfrac{4}{5}, \; r < 1 \qquad a = 96 \times \dfrac{5}{4}$

$a = 120$

$S_{10} = \dfrac{120\left(1 - \frac{4}{5}^{10}\right)}{1 - \frac{4}{5}} = 536 \text{ (3SF)}$

9 Given: $a + ar = ar^2$

a) $\cancel{a}(1+r) = \cancel{a}r^2 \Rightarrow 1 + r = r^2 \Rightarrow r^2 - r - 1 = 0$,

hence r is independent of a

b) $a = 2 \qquad r = \frac{1+\sqrt{5}}{2}, \; r > 0$

$$S_{20} = \frac{2\left(1 - \left(\frac{1+\sqrt{5}}{2}\right)^{20}\right)}{1 - \left(\frac{1+\sqrt{5}}{2}\right)} = 48900 \text{ (3sf)}$$