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Ch-3: Sequences and Series
SET A

BRONZE:

Given: $a_{16} = a + 15d = 25$; $a_{25} = a + 24d = 47.5$

a) $\cancel{a} + 24d = 47.5$
$\cancel{a} + 15d = 25$
$9d = 22.5$

$d = 2.5$ $\Rightarrow a + 15(2.5) = 25$
$\Rightarrow a = -12.5$

b) $S_n = 712.5$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$\Rightarrow 712.5 = \frac{n}{2}(2(-12.5) + (n-1)(2.5))$ $\Rightarrow 1425 = n(-25 + 2.5n - 2.5)$

$\Rightarrow 1425 = -27.5n + 2.5n^2$ $\Rightarrow 2.5n^2 - 27.5n - 1425 = 0$
$(n-30)(n+19) = 0$
$\boxed{n = 30}$, $n > 0$

SILVER:

$p + 2p + 3p + \dots + 200$ $\quad S_n = 1700$

$\Rightarrow a = p \quad d = p(2p - p) \quad l = 200 \quad S_n = \frac{n}{2}(a + l) \quad n = ?$

$\Rightarrow 200 = a + (n-1)d \Rightarrow 200 = p + (n-1)p \Rightarrow 200 = \cancel{p} + pn - \cancel{p}$

$\boxed{n = \frac{200}{p}}$ $\Rightarrow 1700 = \frac{\frac{200}{p}}{2}(p + 200)$ $\Rightarrow 1700 = \frac{100}{p}(p + 200)$

$\Rightarrow 1700p = 100p + 20000$ $\Rightarrow 16\cancel{00}p = 200\cancel{00}$ $\boxed{p = 12.5}$

GOLD:

$a = 3k + 2 \quad a_2 = 5k + 7 \quad a_3 = 7k + 12$

$d = 5k + 7 - 3k - 2 = 2k + 5$

$0 < a + 59d \Rightarrow 3k + 2 + 59(2k + 5) > 0$
$\Rightarrow 3k + 118k + 2 + 295 > 0$
$\Rightarrow 121k + 297 > 0$
$\Rightarrow k > -\frac{297}{121} = k > -\frac{27}{11}$

Set B

BRONZE:

$$\sum_{r=8}^{\infty} 3\times\left(\frac{1}{2}\right)^r \Rightarrow S_\infty = \frac{a}{1-r} \Rightarrow r=8 = 3/256 \rightarrow ar^8$$

$$r=9 = 3/512 \rightarrow ar^9$$

$$r=10 = 3/1024$$

$$\Rightarrow \frac{ar^9}{ar^8} = r = \frac{\cancel{3}}{512}\times\frac{256}{\cancel{3}} = \frac{1}{2}$$

$$S_8 = \frac{3\left(1-(1/2)^8\right)}{1-\frac{1}{2}} = \frac{765}{128} \qquad S_\infty = \frac{a}{1-r} = \frac{3}{0.5} = 6$$

$$\Rightarrow 6 - \frac{765}{128} = \frac{3}{128}$$

SILVER:

$$\sum_{r=8}^{k}(2r-5) = 299 \Rightarrow a = 2(8)-5 = 11 \quad l = 2k-5$$

$$S_n = 299 \quad S_n = \frac{n}{2}(a+l) \Rightarrow 299 = \frac{n}{2}(11+2k-5)$$

$$n = k-8+1 = k-7. \quad \Rightarrow 598 = k-7(6+2k) \Rightarrow 598 = 6k+2k^2-42-14k$$

$$\Rightarrow 2k^2-8k-640 = 0$$

$$\Rightarrow (k-20)(k+16) = 0 \quad \boxed{k=20},\ k>0$$

GOLD:

$$\sum_{r=1}^{16}(3r+2^r+k) = 131550 \qquad a_{16} = 3+15(3) = 48$$

$$\sum_{r=1}^{16} 3r = \frac{n}{2}(a_1+l) \Rightarrow a=3 \quad l=48 \Rightarrow \frac{16}{2}(3+48) = 408$$

$$\sum_{r=1}^{16} 2^r = \frac{a(r^n-1)}{r-1} = \frac{2(2^{16}-1)}{2-1} = 131070$$

$$\sum_{r=1}^{16} k = 16k \Rightarrow 408 + 131070 + 16k = 131550$$

$$k = \frac{72}{16} = \frac{9}{2}$$