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3.5 Sum to infinity

① a) Given: $2 + 0.2 + 0.02 + \dots +$ is convergent.

$r = \frac{0.2}{2} = 0.1$ $\quad |r| = 0.1 < 1$, therefore it is convergent

b) $a = 2 \quad r = 0.1 \quad S_\infty = \frac{a}{1-r} = \frac{2}{1-0.1} = \frac{20}{9}$

② a) i) $5 + 2 + 0.8 + 0.32 + \dots \quad |r| = \frac{2}{5} = 0.4 < 1$, it is convergent

ii) $S_\infty = \frac{5}{1-0.4} = \frac{25}{3} \quad [a = 5 \quad r = 0.4]$

b) i) $0.1 + 0.3 + 0.9 + 1.2 + \dots \quad |r| = \frac{0.3}{0.1} = 3 > 1$, not convergent

c) i) $81 - 27 + 9 - 3 + \dots \quad |r| = -\frac{27}{81} = \frac{1}{3} < 1$, it is convergent

ii) $S_\infty = \frac{81}{1+\frac{1}{3}} = \frac{243}{4} \quad \left[a = 81 \quad r = -\frac{1}{3}\right]$

d) $20 + 18 + 16 + 14 + \dots$ Not convergent as the series is arithmetic

③ Given: $u_5 = ar^4 = 2.4576 \qquad u_7 = ar^6 = 1.572864$

a) $\frac{ar^6}{ar^4} = \frac{1.572864}{2.4576} \Rightarrow r^2 = \frac{16}{25} \qquad r = \pm\frac{4}{5} \quad |r| = 0.8 < 1$

So, it is convergent

b) $S_\infty = \frac{a}{1-r}$; $\quad r = 0.8 \quad a = 6 \Rightarrow \frac{6}{1-0.8} = 30$

$r = -0.8 \quad a = 6 \Rightarrow \frac{6}{1.8} = \frac{10}{3}$

④ Given: $a=8$ $S_\infty=25$

a) $S_\infty=\frac{a}{1-r}$ $25=\frac{8}{1-r}$ $25-25r=8$ $\boxed{r=\frac{17}{25}}$

b) Given: $S_\infty=40$ $r=\frac{1}{5}$

$S_\infty=\frac{a}{1-r} \Rightarrow 40=\frac{a}{1-\frac{1}{5}} \Rightarrow \boxed{a=32}$

⑤ Given: $S_4=80$ $S_\infty=81$

a) $\frac{a(1-r^n)}{1-r}=80$ $\frac{a}{1-r}=81$ $a=81(1-r)$

$\frac{81\cancel{(1-r)}(1-r^n)}{\cancel{1-r}}=80$ $1-r^4=\frac{80}{81}$ $1-\frac{80}{81}=r^4$

$r=\pm\frac{1}{3}$

b) $r>0$, $a=81\left(1-\frac{1}{3}\right)=54$

⑥ Given: $a=4$ $S_\infty=12$

$S_\infty=\frac{a}{1-r}$ $12=\frac{4}{1-r} \Rightarrow 12(1-r)=4 \Rightarrow 12-12r=4$ $r=\frac{8}{12}=\frac{2}{3}$

⑦ a) $|r|<1$

b) Given: $a=3/8$ $r=1/2$ $S_\infty=\frac{a}{1-r}$

$=\frac{3}{8(1-1/2)}=\frac{3}{4}$

⑧ Given: $a=4$ $S_\infty=20\times S_4$

a) $S_\infty=\frac{a}{1-r}=\frac{4}{1-r}$ | $20\times S_4=20\times\frac{a(1-r^4)}{1-r}$

$\Rightarrow \frac{4}{\cancel{1-r}}=\frac{20\times 4(1-r^4)}{\cancel{1-r}} \Rightarrow 4=80(1-r^4) \Rightarrow \frac{1}{20}=1-r^4 \Rightarrow r^4=\frac{19}{20}$

$r=0.9873$

b) $u_5 = ar^4 = (4)(0.9873)^4 = 3.8$

9) Given: $a = 80 \quad S_\infty = 120 \Rightarrow \dfrac{a}{1-r} = 120$

a) $\dfrac{80}{1-r} = 120 \Rightarrow 120 - 120r = 80 \Rightarrow r = \dfrac{1}{3}$

b) $u_4 - u_5 = ar^3 - ar^4 \Rightarrow 80\left(\frac{1}{3}\right)^4 - 80\left(\frac{1}{3}\right)^3 = 1.98$

c) $S_6 = \dfrac{a(1-r^6)}{1-r} = \dfrac{80\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}} = 119.8$ (1dp)

10) Given: $ar = 12/5 \quad S_\infty = 10$

a) $ar = 12/5 \qquad a = \dfrac{12}{5r} \Rightarrow S_\infty = \dfrac{a}{1-r} \Rightarrow 10 = \dfrac{12}{5r(1-r)} \Rightarrow 10 = \dfrac{12}{5r-5r^2}$

$\Rightarrow 50r - 50r^2 = 12 \Rightarrow -50r^2 + 50r - 12 = 0 \Rightarrow 50r^2 - 50r + 12 = 0$

b) $r = \dfrac{3}{5}$ or $r = \dfrac{2}{5}$

c) $a = \dfrac{12}{\cancel{5}\left(\frac{3}{\cancel{5}}\right)} = 4 \qquad a = \dfrac{12}{\cancel{5}\left(\frac{2}{\cancel{5}}\right)} = 6 \qquad a = 6$ or $a = 4$

d) $S_n > 9.99 \Rightarrow \dfrac{6\left(1-\left(\frac{2}{5}\right)^n\right)}{1-\frac{2}{5}} > 9.99 \Rightarrow 30\left(1-(0.4)^n\right) > 29.97$

$\Rightarrow \left(1-(0.4)^n\right) > 0.999 \qquad (0.4^n) < 1 - 0.999$

$\Rightarrow 0.4^n < 0.001 \qquad n < \dfrac{\log(0.001)}{\log(0.4)} \qquad n < 7.5$

$\boxed{n = 8}$