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14.5

① a) $4\log_7 2$

(power law)

$\hookrightarrow \log_7(2^4)$

$= \log_7(16)$.

b) $\log_3 16 - \log_3 2$

(division law)

$\hookrightarrow \log_3\left(\frac{16}{2}\right)$

$= \log_3(8)$.

c) $\log_5 6 + \log_5 7$

(multiplication law)

$\hookrightarrow \log_5(6\times 7)$

$= \log_5(42)$.

d) $3\log_4 5 + 2\log_4 8$

(Power law)

$\hookrightarrow \log_4(5^3) + \log_4(8^2)$

$= \log_4(125\times 64)$

$= \log_4(8000)$.

e) $3\log_7 4 - 4\log_7 3$

(Power law)

$\hookrightarrow \log_7(4^3) - \log_7(3^4)$

(division law)

$\hookrightarrow \log_7(64\div 81)$

$= \log_7\left(\frac{64}{81}\right)$.

f) $(\log_6 3 + \log_6 4) - \log_6 5$

(multiplication law)

$\hookrightarrow \log_6(3\times 4) - \log_6 5$

$\hookrightarrow \log_6(12) - \log_6 5$

$= \log_6\left(\frac{12}{5}\right)$.

② a) $\log_{10} 5 + \log_{10} 20$

(multiplication law)

$\hookrightarrow \log_{10}(5\times 20)$

$= \log_{10}(100)$

(base law)

$10^x = 100$

$x = 2$.

b) $4\log_3 2 + \log_3 4$

(Power law)

$\log_3(2^4) + \log_3 4$

(multiplication law)

$\hookrightarrow \log_3(16\times 4)$

$= \log_3(64)$

(base law)

$\hookrightarrow 8^x = 64$

$x = 2$.

c) $\log_2 48 - \log_2 6$

(division law)

$\hookrightarrow \log_2(48\div 6)$

$= \log_2(8)$

(base law)

$\hookrightarrow 2^x = 8$

$x = 3$.

d) $\log_5 4 - \log_5 20$

(division law)

$\hookrightarrow \log_5(4\div 20)$

$= \log_5\left(\frac{1}{5}\right)$

(base law)

$\hookrightarrow 5^x = \frac{1}{5}$

$x = -1$.

e) $2\log_3 5 + 4\log_3 2 - 2\log_3 10$

(Power law)

$\hookrightarrow \log_3(5^2) + \log_3(2^4) - \log_3(10^2)$

$= (\log_3(25) + \log_3(16)) - \log_3(100)$

(multiplication law)

$\log_3(25\times 16) - \log_3(100)$

$\log_3(400) - \log_3(100)$

(division law) $\log_3(4)$

(base law) $\rightarrow 8^x = 4 \rightarrow x = \frac{2}{3}$

(3) a) $\log_a a^3$

(Power law)

↳ $3\log_a a$ ← $\log_a(a)$ equals to 1 as the logarithm of a number to its own base is always 1

$= 3(1)$

$= 3$

b) $\log_a(x^2y^3)$

(multiplication law)

↳ $\log_a(x^2) + \log_a(y^3)$

(Power law)

↳ $2\log_a x + 3\log_a y$.

c) $\log_a\left(\frac{x^3}{z^4}\right)$

(division law)

↳ $\log_a x^3 - \log_a z^4$

(Power law)

↳ $3\log_a x - 4\log_a z$.

d) $\log_a\left(\frac{x\sqrt{y}}{z^2}\right)$

(division law)

↳ $\log_a(x\sqrt{y}) - \log_a z^2$

↓ (multiplication law)

$= (\log_a x + \log_a y^{1/2}) - \log_a z^2$

↳ Power law

$\log_a x + \frac{1}{2}\log_a y - 2\log_a z$.

e) $\log_a(\sqrt[3]{ax^2yz^4})$

$\log_a(a^{1/3}x^{2/3}y^{1/3}z^{4/3})$

(multiplication law)

↳ $\log_a(a^{1/3}) + \log_a(x^{2/3}) + \log_a(y^{1/3}) + \log_a(z^{4/3})$

(Power law)

↳ $\frac{1}{3}\underbrace{\log_a a}_{=1} + \frac{2}{3}\log_a x + \frac{1}{3}\log_a y + \frac{4}{3}\log_a z$

$= \frac{1}{3} + \frac{2}{3}\log_a x + \frac{1}{3}\log_a y + \frac{4}{3}\log_a z$.

f) $\log_a\left(\frac{a^3x^4y}{\sqrt{z}}\right)$

(division law)

↳ $\log_a(a^3x^4y) - \log_a(z^{1/2})$

(multiplication law)

↳ $\log_a(a^3) + \log_a(x^4) + \log_a y - \log_a(z^{1/2})$

(Power law)

↳ $\underbrace{3\log_a(a)}_{3(1)=3} + 4\log_a x + \log_a y - \frac{1}{2}\log_a z$

$= 3 + 4\log_a x + \log_a y - \frac{1}{2}\log_a z$.

④ a) $\log_6 4 + \log_6 x = 2$

(multiplication law)

↳ $\log_6 (4x) = 2$

(base law)

↳ $6^2 = 4x$

$36 = 4x$

$\underline{x = 9}$.

b) $2\log_8 6 = \log_8 x - 1$

(power law)

↳ $\log_8 (6^2) = \log_8 x - 1$

(division law)

↳ $\log_8 (36) - \log_8 (x) = -1$

(base law) $\log_8 \left(\frac{36}{x}\right) = -1$

↳ $8^{-1} = \frac{36}{x}$

$\frac{1}{8} = \frac{36}{x}$

$\underline{x = 288}$.

c) $2\log_6 x = 2 + \log_6 4$

(power law)

↳ $\log_6 x^2 = 2 + \log_6 4$

(division law)

↳ $\log_6 x^2 - \log_6 4 = 2$

$\log_6 \left(\frac{x^2}{4}\right) = 2$

(base law)

↳ $6^2 = \frac{x^2}{4}$

$36 \times 4 = x^2$

$\sqrt{144} = x^2$

$\underline{x = 12}$.

d) $2\log_4 (x-1) - \log_4 9 = 1$

(power law)

↳ $\log_4 (x-1)^2 - \log_4 9 = 1$

$\log_4 (x^2 - 2x + 1) - \log_4 9 = 1$

(division law)

↳ $\log_4 \left(\frac{x^2-2x+1}{9}\right) = 1$

(base law)

↳ $4^1 = \frac{x^2-2x+1}{9}$

$36 = x^2 - 2x + 1$

$x^2 - 2x - 35 = 0$

$x = 7$ or ~~$x = -5$~~ (can't be negative)

(5) $3\log_5 4 = 2 - 2\log_5 x$

(power law)

↳ $\log_5(4^3) = 2 - \log_5(x^2)$

(multiplication law)

↳ $\log_5(64) + \log_5 x^2 = 2$

$\log_5(64x^2) = 2$

(base law)

↳ $5^2 = 64x^2$

$\frac{25}{64} = x^2$

$x = \frac{5}{8}$

(6) $\log_5 x + 2\log_5 x = 4$

(power law)

↳ $\log_5 x + \log_5 x^2 = 4$

(multiplication law)

↳ $\log_5(x^3) = 4$

(base law)

↳ $5^4 = x^3$

$x = 5^{4/3}$ (separate the exponents into two)

$x = 5^1 \times 5^{1/3}$

$x = 5 \times \sqrt[3]{5}$

$x = 5(\sqrt[3]{5})$

(7) $\log_3(2x+5) - \log_3(4x-1) = 2$

(division law)

↳ $\log_3\left(\frac{2x+5}{4x-1}\right) = 2$

(base law)

↳ $3^2 = \frac{2x+5}{4x-1}$

$9(4x-1) = 2x+5$

$36x - 9 = 2x + 5$

$36x - 2x = 5 + 9$

$34x = 14$

$x = \frac{7}{17}$

(8) a) $\log_2(x+5) - 2\log_2(x-1) = 1$

(power law)

↳ $\log_2(x+5) - \log_2(x-1)^2 = 1$

$\log_2(x+5) - \log_2(x^2-2x+1) = 1$

(division law)

↳ $\log_2\left(\frac{x+5}{x^2-2x+1}\right) = 1$

(base law)

↳ $2^1 = \frac{x+5}{x^2-2x+1}$

$2(x^2-2x+1) = x+5$

$2x^2 - 4x + 2 = x + 5$

$2x^2 - 4x - x + 2 - 5 = 0$

$2x^2 - 5x - 3 = 0$

b) using $2x^2 - 5x - 3$

↳ $(x-3)(2x+1)$

$x = 3$ or ~~$x = -\frac{1}{2}$~~ (can't be negative)