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# Exponentials and logarithms
# Problem solving set B

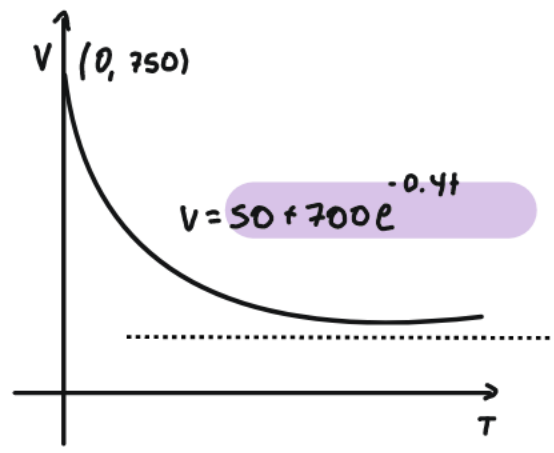
## Bronze

a) $t = 0$

$V = 50 + 700e^{-0.4(0)}$

$V = 750$

Asymptote $= 50$




b) $t = 5$

$V = 50 + 700e^{-0.4(5)}$

$V = 144.73$

c) $100 = 50 + 700e^{-0.4t}$

$50 = 700e^{-0.4t}$

$\frac{1}{14} = e^{-\frac{2}{5}t}$

$\ln \frac{1}{14} = \ln e^{-\frac{2}{5}t}$

$\ln \frac{1}{14} = -\frac{2}{5}t$

$-\frac{5}{2} \ln \frac{1}{14} = t$

## Silver

a) $t = 11$

$C = e^{\frac{1}{4}(11)} - 1$

$C = 14.6426^{\times 1000}$

$C = 14642.6$

$C = 14643$

b) $C = e^{\frac{1}{4}t} - 1$

$\frac{dC}{dt} = \frac{1}{4} e^{\frac{1}{4}t}$

c) $12 = \frac{1}{4} e^{\frac{1}{4}t}$

$48 = e^{\frac{1}{4}t}$

$\ln 48 = \ln e^{\frac{1}{4}t}$

$\ln 48 = \frac{1}{4}t$

$4 \ln 48 = t$

## Gold

a) Because the temperature of the soup decreases over time so k must be negative

b) $70 = ae^{k \cdot 0} + 25$

$70 = a + 25$

$a = 45$

c) $55 = 45e^{k5} + 25$

$30 = 45e^{k5}$

$\frac{2}{3} = e^{k5}$

$\ln \frac{2}{3} = \ln e^{k5}$

$\ln \frac{2}{3} = 5k$

$\frac{1}{5} \ln \frac{2}{3} = k$

d) $T = ae^{-kt} + 25$

$\frac{dT}{dt} = kae^{kT}$

$\frac{dT}{dt} = \frac{1}{5} \ln\left(\frac{2}{3}\right) \times 45e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t}$

$= 9\ln\left(\frac{2}{3}\right)e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t}$

Decreasing rate of $1.5^{\circ}C \Rightarrow \frac{dT}{dt} = -1.5$ ← decreasing

$9\ln\left(\frac{2}{3}\right)e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t} = -1.5$

$e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t} = -1.5 \div \left(9\ln\left(\frac{2}{3}\right)\right)$

make t the subject

$e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t} = 0.41105$

$\ln e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t} = \ln(0.41105)$

$\frac{1}{5}\ln\left(\frac{2}{3}\right)t = \ln(0.41105)$

$\ln\left(\frac{2}{3}\right)t = \ln(0.41105) \times 5$

$t = \frac{\ln(0.41105) \times 5}{\ln\left(\frac{2}{3}\right)}$

$t = 10.96$ mins