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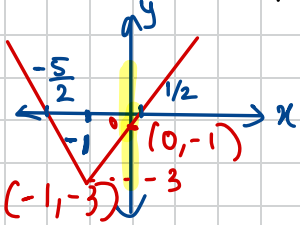
2.7 : Solving modulus problem

① $f(x) = 2|x+1| - 3, \quad x \in \mathbb{R}$ → $\begin{matrix} x & 0 & & \\ y & 2 & 4 & -3 \end{matrix}$

a) 

$2(x+1) - 3 = 0$ or $-2x - 2 = 3$

$2x + 2 = 3$ $\qquad -2x = 5$

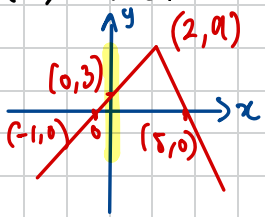
$x = \frac{1}{2}$ $\qquad x = -\frac{5}{2}$

b) Range : $f(x) \geq -3$

c) $f(x) = x + 4$ i) $2(x+1) - 3 = x + 4 \Rightarrow 2x - 1 = x + 4 \Rightarrow x = 5$

ii) $-(2x+1) - 3 = x + 4 \Rightarrow -2x - 2 - 3 = x + 4 \Rightarrow -3x = 9 \Rightarrow x = -3$

② $g(x) = 9 - 3|x - 2|, \quad x \in \mathbb{R}$ → $\begin{matrix} x & +2 & & \\ y & x-3 & \xi & +9 \end{matrix}$

a) 

$9 - 3(x-2) = 0$ or $9 + 3(x-2) = 0$

$9 - 3x + 6 = 0$ $\qquad 9 + 3x - 6 = 0$

$-3x = -15$ $\qquad 3x = -3$

$x = 5$ $\qquad x = -1$

b) Range: $g(x) \leq 9$

c) $g^{-1}(x)$ is not an one-to-one function.

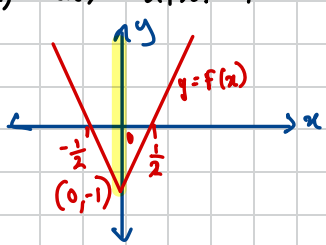
d) $g(x) \leq 3 \Rightarrow 9 - 3(x-2) \leq 3 \Rightarrow 9 - 3x + 6 - 3 \leq 0 \Rightarrow -3x + 12 \leq 0$

$x \geq 4$

$9 + 3(x-2) \leq 3 \Rightarrow 9 + 3x - 6 \leq 3 \Rightarrow 3x \leq 0 \quad x \leq 0$

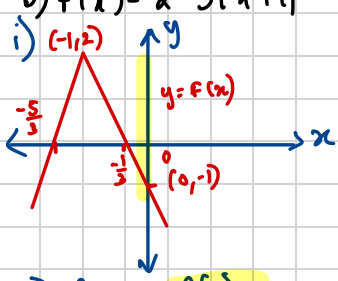
$\{x : x \leq 0\} \cup \{x : x \geq 4\}$

③ a) $f(x) = 2|x| - 1$

i) 

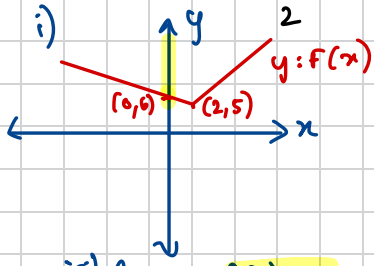
ii) Range: $f(x) \geq -1$

b) $f(x) = 2 - 3|x+1|$

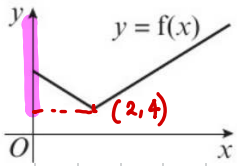
i) 

ii) Range: $f(x) \leq 2$

c) $f(x) = 5 + \frac{|x-2|}{2}$

i) 

iii) Range: $f(x) \geq 5$

④ 

$f(x) = 3|2-x| + 4, \quad x \geq 0$

a) Range: $f(x) \geq 4$

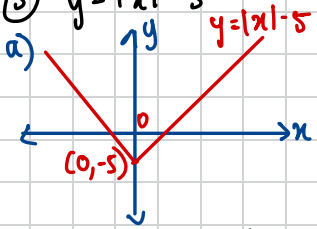
b) $f(x) = x + 20 \Rightarrow$ i) $3(2-x)+4 = x+20$

$\Rightarrow 6 - 3x + 4 = x + 20 \Rightarrow 4x = -10 \quad x = -\frac{5}{2}$, $x \geq 0$

ii) $-3(2-x) + 4 = x + 20 \Rightarrow -6 + 3x + 4 = x + 20 \Rightarrow 2x = 22 \Rightarrow x = 11$

c) $f(x) = k \Rightarrow f(0) = 10 \Rightarrow 4 < k \leq 10$

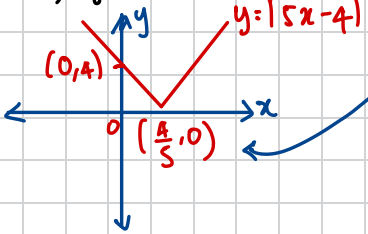
⑤ $y = |x| - 5$

a) 

b) For $x \geq 5$, $|x| - 5 = |x - 5|$; $x < 5$, $|x| - 5 < |x - 5|$

So, $|x| - 5 \leq |x - 5|$ for all real values of x

⑥ a) $y = |5x - 4|$



$5x - 4 = 0$

$x = \frac{4}{5}$

b) $|5x - 4| > 3 - 2x$

$5x - 4 > 3 - 2x$ or $-5x + 4 > 3 - 2x$

$\Rightarrow 7x > 7$ $\quad 1 > 3x$

$x > 1$ or $\frac{1}{3} > x$

$\{x : x < \frac{1}{3}\} \cup \{x : x > 1\}$

c) $|5x - 4| > \frac{8}{5} - 2x \Rightarrow$ i) $5x - 4 > \frac{8}{5} - 2x \quad : 25x - 20 > 8 - 10x$

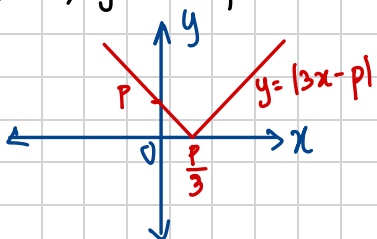
$\Rightarrow 35x > 28 \quad x > \frac{4}{5}$

ii) $-5x + 4 > \frac{8}{5} - 2x \Rightarrow -25x + 20 > 8 - 10x \Rightarrow 15x < 12$

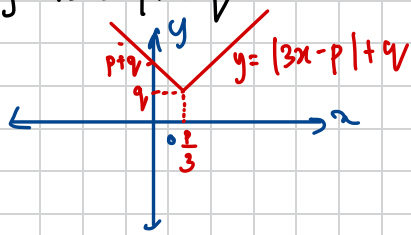
$x < \frac{4}{5}$

$\Rightarrow \{x : x > \frac{4}{5}\} \cup \{x : x < \frac{4}{5}\}$

⑦ a) i) $y = |3x - p|$



ii) $y = |3x - p| + q$



b) $|3x-p|+q = 2x+8$ has solution $x=0$ and $x=k$

$\Rightarrow |3(0)-p|+q = 2(0)+8 \Rightarrow p+q=8$ — (1) $\boxed{q=8-p}$

$\Rightarrow |3k-p|+q = 2k+8 \Rightarrow 3k-p+\cancel{8}-p = 2k+\cancel{8}$

$\Rightarrow 3k-2p = 2k \Rightarrow 3k-2k = 2p \Rightarrow \boxed{k=2p}$