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14.7 Working with natural logarithms

1.

a) $e^x = 8$

$\ln(e^x) = \ln 8$

$x = \ln 8$

b) $e^{\frac{1}{2}x} = 6$

$\ln(e^{\frac{1}{2}x}) = \ln 6$

$\frac{1}{2}x = \ln 6$

$x = 2\ln 6$ or $x = \ln 36$ $(\ln(6)^2)$

c) $e^{x+5} = 81$

$\ln(e^{x+5}) = \ln 81$

$x + 5 = \ln 81$

$x = -5 + \ln 81$

d) $e^{5x-1} = 23$

$\ln(e^{5x-1}) = \ln 23$

$5x - 1 = \ln 23$

$5x = 1 + \ln 23$

$x = \dfrac{1 + \ln 23}{5}$

$x = \frac{1}{5} + \frac{1}{5}\ln 23$

e) $4e^{1-x} = 29$

$\ln(4e^{1-x}) = \ln 29$

$4(1-x) = \ln 29$

$1 - x = \ln \frac{29}{4}$

$x = 1 - \ln\frac{29}{4}$

f) $\frac{2}{3}e^{3x} = 7$

$\ln\left(\frac{2}{3}e^{3x}\right) = \ln 7$

$\frac{2}{3}(3x) = \ln 7$

$3x = \ln\frac{21}{2}$

$x = \frac{1}{3}\ln\frac{21}{2}$

2.

a)
$$\ln x = 11$$
$$e^{\ln x} = e^{11}$$
$$x = e^{11}$$

b)
$$\ln 6x = 3$$
$$e^{\ln 6x} = e^{3}$$
$$6x = e^{3}$$
$$x = \frac{1}{6}e^{3}$$

c)
$$\ln(3-5x) = -1$$
$$e^{\ln(3-5x)} = e^{-1}$$
$$3-5x = e^{-1}$$
$$3-5x = \frac{1}{e}$$
$$5x = 3-\frac{1}{e}$$
$$5x = \frac{3e-1}{e}$$
$$x = \frac{3e-1}{5e}$$

d)
$$\ln(5-2x) = \frac{2}{5}$$
$$e^{\ln(5-2x)} = e^{\frac{2}{5}}$$
$$5-2x = e^{\frac{2}{5}}$$
$$2x = 5-e^{\frac{2}{5}}$$
$$x = \frac{5-e^{\frac{2}{5}}}{2}$$

e)
$$2\ln(3x-5) = 7$$
$$2e^{\ln(3x-5)} = e^{7}$$
$$e^{\ln(3x-5)} = e^{\frac{7}{2}}$$
$$3x-5 = e^{\frac{7}{2}}$$
$$3x = 5+e^{\frac{7}{2}}$$
$$x = \frac{5+e^{\frac{7}{2}}}{3}$$

f)
$$\ln(x^2-9x+21) = 0$$

4,5

$$e^{\ln(x^2-9x+21)} = e^{0}$$
$$x^2-9x+21 = e^{0}$$
$$x^2-9x+21 = 1 \qquad e^{0}=1$$
$$x^2-9x+20 = 0$$
$$x=5 \qquad x=4$$

3.
$$e^{2x} - 5e^{x} + 4 = 0$$

let y be e^{x}:

$$y^2 - 5y + 4 = 0$$
$$y = 4 \qquad y = 1$$

$$e^{x} = 4 \qquad e^{x} = 1$$
$$\ln(e^{x}) = \ln 4 \qquad \ln(e^{x}) = \ln 1$$
$$x = \ln 4 \qquad x = \ln 1$$
$$x = 0$$

4. $2e^{4x} - 11e^{2x} + 14 = 0$

let y be e^{2x}:

$2y^2 - 11y + 14 = 0$

$y = \frac{7}{2}$ $\quad y = 2$

$e^{2x} = \frac{7}{2}$ $\quad e^{2x} = 2$

$\ln(e^{2x}) = \ln\frac{7}{2}$ $\quad \ln(e^{2x}) = \ln 2$

$2x = \ln\frac{7}{2}$ $\quad 2x = \ln 2$

$x = \frac{1}{2}\ln\frac{7}{2}$ $\quad x = \frac{1}{2}\ln 2$

5. $e^x - 12 = e^{\frac{x}{2}}$

$e^x - 12 - e^{\frac{x}{2}} = 0$

$e^x - e^{\frac{x}{2}} - 12 = 0$

let y be $e^{\frac{x}{2}}$

$y^2 - y - 12 = 0$

$y = 4$ $\quad y = -3$

$e^{\frac{x}{2}} = 4$ $\quad e^{\frac{x}{2}} = -3$

$\ln(e^{\frac{x}{2}}) = \ln 4$ $\quad \ln(e^{\frac{x}{2}}) = \ln -3$

$\frac{x}{2} = \ln 4$ $\quad \frac{x}{2} = \ln -3 \Rightarrow$ no real solutions

$x = 2\ln 4$

$x = \ln(4)^2$

$x = \ln 16$

6. $2e^{4x} + 15e^{-4x} = 13$

$2e^{4x} + \frac{15}{e^{4x}} = 13$

$2e^{4x} + \frac{15}{e^{4x}} - 13 = 0$

$\times e^{4x}$ on left side

$2e^{8x} + 15 - 13e^{4x} = 0$
$2e^{8x} - 13e^{4x} + 15 = 0$

let y be e^{4x}:

$2y^2 - 13y + 15 = 0$
$y = 5$ $\quad$ $y = \frac{3}{2}$

$e^{4x} = 5$
$\ln(e^{4x}) = \ln 5$
$4x = \ln 5$
$x = \frac{1}{4}\ln 5$

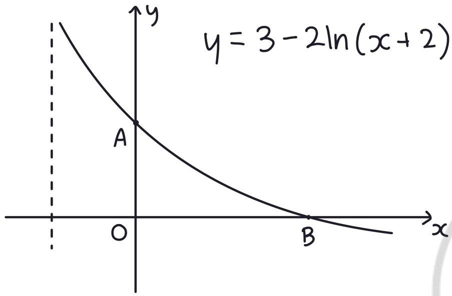
$e^{4x} = \frac{3}{2}$
$\ln(e^{4x}) = \ln\frac{3}{2}$
$4x = \ln\frac{3}{2}$
$x = \frac{1}{4}\ln\frac{3}{2}$

7. $e^{1-6x} = 4(12^x)$

$\ln(e^{1-6x}) = \ln 4(12^x)$
$1 - 6x = \ln 4(12^x)$
$1 - 6x = \ln 4 + x\ln 12$
$1 - \ln 4 = x\ln 12 + 6x$
$1 - \ln 4 = x(\ln 12 + 6)$
$x = \frac{1 - \ln 4}{\ln 12 + 6}$

$x = \frac{1 - \ln 4}{6 + \ln 12}$

8.



a) When $x = 0$

$$y = 3 - 2\ln(0+2)$$
$$y = 3 - 2\ln 2$$
$$y = 3 - \ln 4$$

$$A = (0,\ 3 - \ln 4)$$

b) When $y = 0$

$$3 - 2\ln(x+2) = 0$$
$$-2\ln(x+2) = -3$$
$$\ln(x+2) = \frac{-3}{-2}$$
$$\ln(x+2) = \frac{3}{2}$$
$$e^{\ln(x+2)} = e^{\frac{3}{2}}$$
$$x + 2 = e^{\frac{3}{2}}$$
$$x = e^{\frac{3}{2}} - 2$$
$$x = -2 + e^{\frac{3}{2}}$$

$$B = (-2 + e^{\frac{3}{2}},\ 0)$$

9.

a) $T = 31e^{-0.05x} + 7$

When $x = 0$:

$T = 31e^{-0.05(0)} + 7$

$T = 31 \times 1 + 7$

$T = 31 + 7$

$T = 38$

T intercept $= (0, 38)$

When $T = 0$:

$0 = 31e^{-0.05x} + 7$

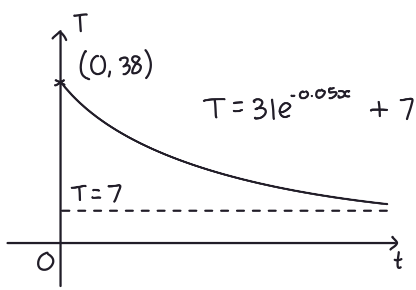
$31e^{-0.05x} = -7$

$e^{-0.05x} = \frac{-7}{31}$

$-0.05x = \ln\left(\frac{-7}{31}\right)$

$x = \frac{\ln\left(-\frac{7}{31}\right)}{-0.05} \implies$ no real solution $\therefore x = \infty$

Sketch graph:



b) According to the graph, when $x = \infty$, $T \to 7$

So 7 represents the minimum temperature of the bath water

c) when $T = 25$:

$$25 = 31e^{-0.05x} + 7$$
$$31e^{-0.05x} = 18$$
$$e^{-0.05x} = \frac{18}{31}$$
$$-0.05x = \ln\left(\frac{18}{31}\right)$$
$$x = \frac{\ln\left(\frac{18}{31}\right)}{-0.05}$$

$$x \approx 10.9 \text{ minutes}$$

So Jack will first add more hot water after 10.9 minutes