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Ch-2: Functions and graphs
Set A

BRONZE:

$f: x \mapsto 3 - 2x^3 \qquad g: x \mapsto \frac{2}{x} - 5, \quad x \in R, \ x > 0$

a) $f^{-1}(x) \Rightarrow y = 3 - 2(x)^3 \Rightarrow \frac{3-y}{2} = x^3 \Rightarrow \sqrt[3]{\frac{3-y}{2}} = x \Rightarrow f^{-1}(x) = \sqrt[3]{\frac{3-x}{2}}$

b) $gf: x \mapsto \frac{2}{3-2x^3} - 5 \Rightarrow gf(x) = \frac{2 - 5(3-2x^3)}{3-2x^3} = \frac{2 - 15 + 10x^3}{3-2x^3}$

$\Rightarrow gf(x) = \frac{10x^3 - 13}{3 - 2x^3}$

c) $gf(x) = 1 \Rightarrow 1 = \frac{10x^3-13}{3-2x^3} \Rightarrow 3 - 2x^3 = 10x^3 - 13$

$\Rightarrow 16 = 12x^3 \Rightarrow \frac{4}{3} = x^3 \Rightarrow x = \sqrt[3]{\frac{4}{3}}$

SILVER:

$f(x) = e^{2x} + 2, \ x \in R$

a) $y = e^{2x} + 2 \Rightarrow y - 2 = e^{2x} \Rightarrow \ln(y-2) = \cancel{\ln e}^{2x}$

$\Rightarrow \ln(y-2) = 2x \Rightarrow x = \frac{\ln(y-2)}{2} \Rightarrow f^{-1}(x) = \frac{\ln(x-2)}{2}, x \in R$

$x > 2$

$g(x) = \ln(x+3), \ x \in R, \ x > -3$

b) $fg(x) = e^{2\ln(x+3)} + 2 = \cancel{e}^{\cancel{\ln}(x+3)^2} + 2 \Rightarrow (x+3)^2 + 2, \ fg(x) \geq 2$

c) $fg(x) = 66 \Rightarrow (x+3)^2 + 2 = 66 \Rightarrow (x+3)^2 = 64 \Rightarrow x + 3 = \pm 8$

$\Rightarrow x + 3 = 8 \ \boxed{x = 5}; \quad x + 3 = -8 \ \boxed{\cancel{x = -11}};$

GOLD:

$f: x \mapsto \dfrac{x-5}{x^2+2x-3} - \dfrac{2}{x+3} + 2, \ x \in \mathbb{R}, \ x > 1$

a) $f(x) = \dfrac{x-5}{(x+3)(x-1)} - \dfrac{2}{(x+3)} + 2 \Rightarrow \dfrac{x-5-2(x-1)+2(x^2+2x-3)}{(x+3)(x-1)}$

$\Rightarrow \dfrac{x-5-2x+2+2x^2+4x-6}{(x+3)(x-1)} = \dfrac{2x^2+3x-9}{(x+3)(x-1)} = \dfrac{(2x-3)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)}$

$\Rightarrow f(x) = \dfrac{2x-3}{x-1}$

b) $x > 1$; $x = 2 \Rightarrow y = 1$; $x = 3 \Rightarrow y = 1 \cdot 5$; $\Rightarrow x = 1000$; $y = 1 \cdot 99\ldots$

$f(x) < 2$

c) $y = \dfrac{2x-3}{x-1} \Rightarrow yx - y = 2x - 3 \Rightarrow yx - 2x = y - 3$

$\Rightarrow x(y-2) = y-3 \Rightarrow x = \dfrac{y-3}{y-2} \Rightarrow f^{-1}(x) = \dfrac{x-3}{x-2}$; $x < 2$

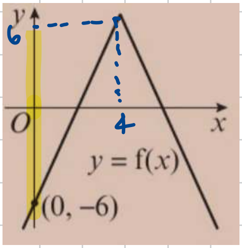
d) $g(x) = 3x^2 - 4 \quad fg(x) = \dfrac{1}{5}$

$\Rightarrow \dfrac{2(3x^2-4)-3}{3x^2-4-1} = \dfrac{1}{5} \Rightarrow \dfrac{6x^2-8-3}{3x^2-5} = \dfrac{1}{5} \Rightarrow \dfrac{6x^2-11}{3x^2-5} = \dfrac{1}{5}$

$\Rightarrow 30x^2 - 55 = 3x^2 - 5 \Rightarrow 27x^2 = 50 \Rightarrow x = \pm\sqrt{\dfrac{50}{27}} = \pm\dfrac{5\sqrt{6}}{9}$

Set B

BRONZE:

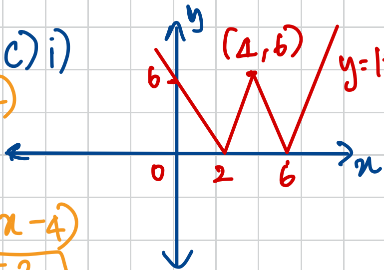


$f(x) = k - 3|x-4| \quad (0,-6)$

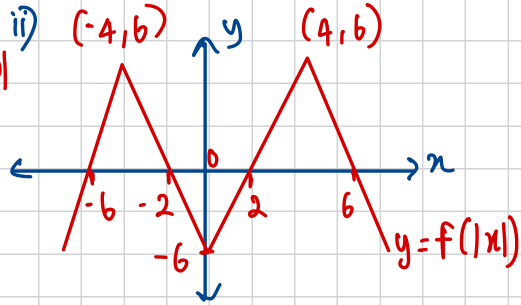
a) $-6 = k - 3|-4| \Rightarrow -6 = k - 3(4) \Rightarrow -6 = k - 12 \quad \boxed{k=6}$

b) $f(x) \leq 6$

c) i)



ii)



$0 = 6 - 3(x-4)$

$\cancel{-}6 = \cancel{-}3(x-4)$

$2 = x-4$

$\boxed{x=6}$ or $-6 = 3(x-4)$

$-2 = x - 4 \quad \boxed{x=2}$

d) $f(x) = \frac{x}{2} + 1$

$\Rightarrow 6 - 3(x-4) = \frac{x}{2} + 1$

$\Rightarrow -3(x-4) = \frac{x}{2} - 5$

$\Rightarrow -6(x-4) = x - 10$

$\Rightarrow -6x + 24 = x - 10$

$\Rightarrow 34 = 7x$

$\Rightarrow x = \frac{34}{7}$

$\Rightarrow 6 + 3(x-4) = \frac{x}{2} + 1$

$\Rightarrow 3(x-4) = \frac{x}{2} - 5$

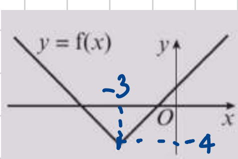
$\Rightarrow 6(x-4) = x - 10$

$\Rightarrow 6x - 24 = x - 10$

$\Rightarrow 5x = +14$

$\Rightarrow x = \frac{14}{5}$

SILVER:



$f(x) = 2|x+3| - 4$

a) $f(x) \geq -4$

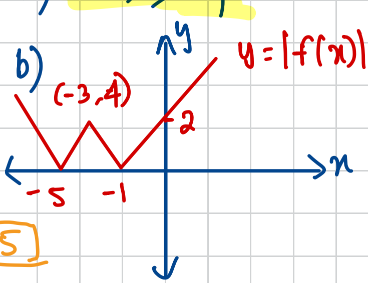
$2(x+3) - 4 = 0$

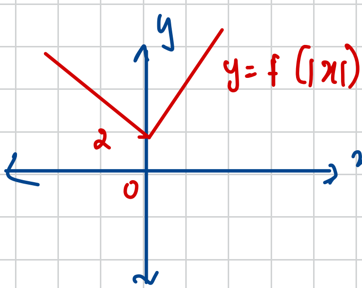
$\cancel{2}(x+3) = \cancel{4}^{2}$

$x + 3 = 2$; $\boxed{x=-1}$

$(x+3) = -2 \quad \boxed{x=-5}$

b)





c) $f(x) \le 4 - 0.5x$

$\Rightarrow 2(x+3) - 4 \le 4 - 0.5x$

$\Rightarrow 2x + 6 - 4 \le 4 - 0.5x$

$\Rightarrow \frac{5}{2}x \le 2 \Rightarrow x \le \frac{4}{5}$

$\Rightarrow -2(x+3) - 4 \le 4 - 0.5x$

$\Rightarrow -2x - 6 - 4 \le 4 - 0.5x$

$\Rightarrow -\frac{3}{2}x \le 14 \Rightarrow x \ge -\frac{28}{3}$

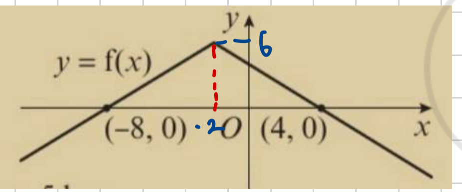
$$\left\{x : -\frac{28}{3} \le x \le \frac{4}{5}\right\}$$

d) $f(x) = k \Rightarrow$ i) 0 solution $\Rightarrow k < -4$

ii) 1 solution $\Rightarrow k = -4$

iii) 2 solution $\Rightarrow k > -4$

GOLD:



$f(x) = a - |x + b|,\ x \in \mathbb{R}$.

Axis symmetry:

$\frac{4+8}{2} = \frac{12}{2} = 6$. $\Rightarrow b = 2$

$\Rightarrow 0 = a - |b - 8| \qquad 0 = a - |b + 4|$

$\Rightarrow (b-8)^2 = (b+4)^2 \Rightarrow \cancel{b^2} - 16b + 64 = \cancel{b^2} + 16 + 8b$

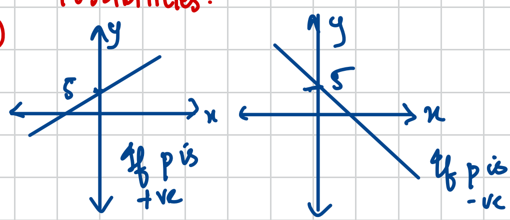
$\Rightarrow 48 = 24b \qquad \boxed{b = 2} \qquad \Rightarrow f(x) = a - |x + 2|$

$f(-8) = 0 \Rightarrow a - |-6| = 0 \Rightarrow a = 6$

b) $f(x) = px + 5$ → let's call this $g(x)$

↳ p represents the gradient (m)

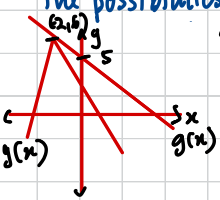
Possibilities:



b) i) If $6 - |x+2| = px + 5$ has 0 solution

↳ Two graphs have no intersection

The possibilities:

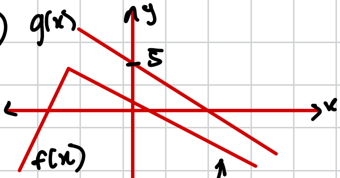


Sub $(-2, 6) \Rightarrow 6 - |-2+2| = p(-2) + 5$

$6 - 0 = -2p + 5$

$\therefore p = -\frac{1}{2} \Rightarrow p > -\frac{1}{2}$ (">" because less steeper cannot equal to -0.5 because otherwise there is one intersection point)

(2)



This part has gradient $=-1$

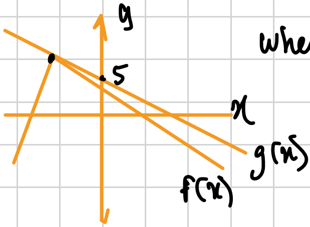
maximum steepness = parallel to the right side of f(x)

$\therefore$ max gradient $=-1$

$\Rightarrow$ $-1 \leq p < -\frac{1}{2}$

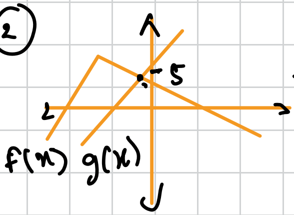
bii) there are 3 possibilities to have 1 intersection point.

(1)



when $p=\frac{-1}{2}$; two graphs intersect at $(-2,6)$

(2)

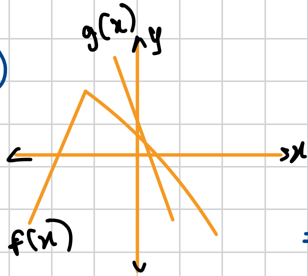


When g(x) is parallel to the left side of f(x), or steeper.

$\rightarrow$ g(x) will only touch the right side of f(x), hence only one intersection point.

$\Rightarrow p \geq 1$

(3)

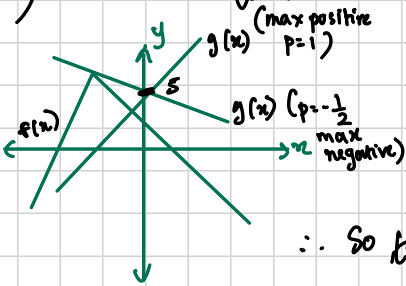


When g(x) is steeper than the right side of f(x). then it will only intersect once.

$\Rightarrow p < -1$ (not $\leq$ because if $p=-1$, then g(x) will be parallel to f(x) which means no intersection point

$\Rightarrow$ $p=-\frac{1}{2}$ or $p \geq 1$ or $p < -1$

biii) For the equation to have two solutions; g(x) must intersect f(x) twice.



For the two graphs to intersect twice. the gradient of g(x) cannot exceed 1, because if $p \geq 1$, then it will be parallel to the left side of f(x), which results in one intersection point only. $\therefore p < 1$

Max negative steepness:

$p=-\frac{1}{2}$, g(x) intersects with f(x) once, at $(-2,6)$

$\therefore$ So to intersect twice, g(x) needs to be flatter. $\Rightarrow p > -\frac{1}{2}$

$\Rightarrow$ $-\frac{1}{2} < p < 1$