

Author: Blinzy Fernandes

This step-by-step solution guide has been created by **Blinzy Fernandes** for educational purposes.

While we have made every effort to ensure the accuracy of the information presented, it is possible that there may be errors or omissions. We encourage users to critically evaluate and verify the content. BF Maths and the author cannot be held responsible for any errors or inaccuracies in this guide.

If you find any mistakes or have any suggestions for improvements, please contact us at bfmathshello@gmail.com. Your feedback is invaluable in helping us maintain the quality and accuracy of our resources. Please specify which exercise and which question in the email.

Thank you for using BF Maths for your maths revision!

Problem Solving Set A Chapter 1

Bronze

$$a) \frac{3n^2 + n - 2}{(n+1)(n-1)} \equiv \frac{A}{n+1} + \frac{B}{n-1}$$

$$\frac{(3n-2)(n+1)}{(n+1)(n-1)} = \frac{3n-2}{n-1}$$

$$b) \frac{3n-2}{n-1} - \frac{1}{n(n-1)}$$

$$\frac{(3n-2)(n) - 1}{n(n-1)}$$

$$\frac{3n^2 - 2n - 1}{n(n-1)} = \frac{(n+1)(3n+1)}{n(n-1)}$$

$$= \frac{3n+1}{n}$$

Silver

$$\frac{n(2n-3)}{(2n-3)(n+2)} - \frac{6}{(n-1)(n+2)}$$

$$\frac{n(2n-3)(n-1) - 6(2n-3)}{(2n-3)(n-2)(n-1)}$$

$$\frac{n(2n^2 - 2n - 3n + 3) - 12n + 18}{(2n-3)(n-2)(n-1)}$$

$$\frac{2n^3 - 5n^2 + 3n - 12n + 18}{(2n-3)(n-2)(n-1)}$$

$$\frac{2n^3 - 5n^2 - 9n + 18}{(2n-3)(n-2)(n-1)}$$

$$\frac{(n+2)(n-3)(2n-3)}{(2n-3)(n-2)(n-1)}$$

$$= \frac{n-3}{n-1}$$

Gold

$$\frac{2n^3 + 3n^2 - 5n - 6}{3n^2 + 5n + 2}$$

$$= \frac{2n^3 + 3n^2 - 5n - 6}{(3n+2)(n+1)}$$

$$\text{When } f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = 0$$

Gold

Hence $(n+1)$ is a factor of $2n^3 + 3n^2 - 5n - 6$

$$\begin{array}{r} 2n^2 + n - 6 \\ n+1 \overline{) 2n^3 + 3n^2 - 5n - 6} \\ \underline{2n^3 + 2n^2} \\ + n - 6 \\ \underline{+ n} \\ - 6 \\ \underline{+ 6} \\ 0 \end{array}$$

$$f(n) = \frac{(2n^2 + n - 6)(n+1)}{(3n+2)(n+1)}$$

$$\begin{array}{r} \frac{2n}{3} - \frac{1}{9} \\ 3n+2 \overline{) 2n^2 + n - 6} \\ \underline{2n^2 + \frac{4n}{3}} \\ - \frac{1}{3}n - 6 \\ \underline{+ \frac{1}{3}n} \\ \phantom{+ \frac{1}{3}n} - \frac{2}{9} \\ \phantom{+ \frac{1}{3}n} \underline{+ \frac{2}{9}} \\ \phantom{+ \frac{1}{3}n} \phantom{+ \frac{2}{9}} - \frac{52}{9} \end{array}$$

$$= An + B + \frac{C}{Dn+1}$$

$$= \frac{2n}{3} - \frac{1}{9} - \frac{\left(\frac{52}{9}\right)}{3n+2}$$

$$= \frac{2n}{3} - \frac{1}{9} - \frac{52}{27n+18}$$

Problem Solving

Set B

Chapter 1

Bronze

$$\frac{39n^2 - 49n + 15}{(3n-2)^2(1-n)} = \frac{A}{1-n} + \frac{B}{3n-2} + \frac{C}{(3n-2)^2}$$

$$A(3n-2)^2 + B(1-n)(3n+2) + C(1-n)$$

When $n=1$

$$5 = A$$

When $n = \frac{2}{3}$

$$-\frac{1}{3} = \frac{1}{3}C$$

$$C = -1$$

When $n=0$

$$15 = 4A - 2B + C$$

$$15 = 20 - 2B - 1$$

$$B = 2$$

$$A = 5 \quad B = 2 \quad C = -1$$

Silver

$$\frac{3n^2 - 2n - 4}{(n-3)(n+2)} = \frac{A}{n-3} + \frac{C}{n+2}$$

$$\begin{array}{r} 3 \\ n^2 - n - 6 \overline{) 3n^2 - 2n + 4} \\ \underline{3n^2 - 3n - 18} \\ + n + 22 \end{array}$$

$$\text{let } \frac{n+22}{(n-3)(n+2)} = \frac{B}{n-3} + \frac{C}{n+2}$$

$$B(n+2) + C(n-3)$$

When $n=-2$

$$20 = -5C$$

$$C = -4$$

When $n=3$

$$25 = 5B$$

$$B = 5$$

$$A = 3 \quad B = 5 \quad C = -4$$

Gold

$$f(n) = \frac{3n^2 + 3}{2n^3 - 5n^2 - 4n + 3}$$

$$f(-1) = \frac{2(-1)^3 - 5(-1)^2 - 4(-1) + 3}{0}$$

Hence $f(-1)$ is a factor

Gold

$$\begin{array}{r} 2n^2 - 7n + 3 \\ n+1 \overline{) 2n^3 - 5n^2 - 4n + 3} \\ \underline{2n^3 + 2n^2} \\ - 7n^2 - 4n \\ - 7n^2 - 7n \\ + 3n + 3 \\ + 3 \\ 0 \end{array}$$

$$f(n) = \frac{3n^2 + 3}{(n+1)(2n^2 - 7n + 3)}$$

$$= \frac{3n^2 + 3}{(n+1)(n-3)(2n-1)}$$

$$= \frac{3(n^2 + 1)}{(n+1)(n-3)(2n-1)}$$

$$= \frac{A}{n+1} + \frac{B}{n-3} + \frac{C}{2n-1}$$

$$A(n-3)(2n-1) + B(n+1)(2n-1) + C(n+1)(n-3)$$

$$= A(2n^2 - 7n + 3) + B(2n^2 + n - 1) + C(n^2 - 2n - 3)$$

Equating them

$$2A + 2B + C = 3$$

$$-7A + B - 2C = 0$$

$$3B - B - 3C = 3$$

$$A = \frac{1}{2} \quad B = \frac{3}{2} \quad C = -1$$

$$= \frac{1}{2(n+1)} - \frac{1}{2n-1} + \frac{3}{2(n-3)}$$