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2-2: Functions and mappings

① a) many to one b) one to one c) one to many d) many to one

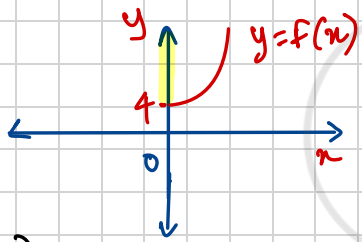
② a) $y = \sqrt{x+2}$ $f: x \mapsto \sqrt{x+2}$, $x \in R$, $x \geq -2$

b) $y = \frac{1}{x+1}$ $f: x \mapsto \frac{1}{x+1}$, $x \in R$, $x \neq -1$

c) $y = \tan x$ $f: x \mapsto \tan x$, $x \in R$, $-90° < x < 90°$

③ a) $f(x) = (x+2)^2$, $x \in R$, $x \geq 0$

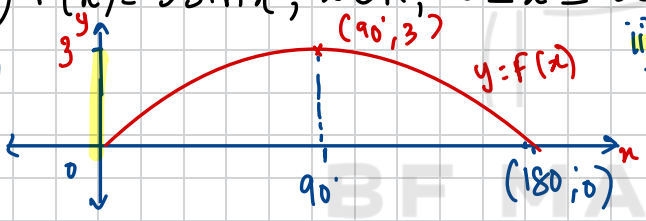
i)



ii) Range: $f(x) \geq 4$ iii) one-to-one

b) $f(x) = 3\sin x$, $x \in R$, $0 \leq x \leq 180°$

i)



ii) Range: $0 \leq f(x) \leq 3$

iii) many-to-one

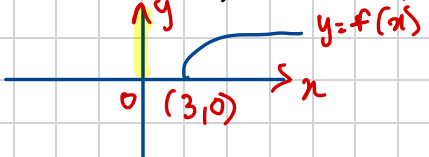
c) $f(x) = 2e^x$, $x \in R$, $x \geq 0$

i)

ii) Range: $f(x) \geq 2$ iii) one-to-one

d) $f(x) = \sqrt{x-3}$, $x \in R$, $x \geq 3$

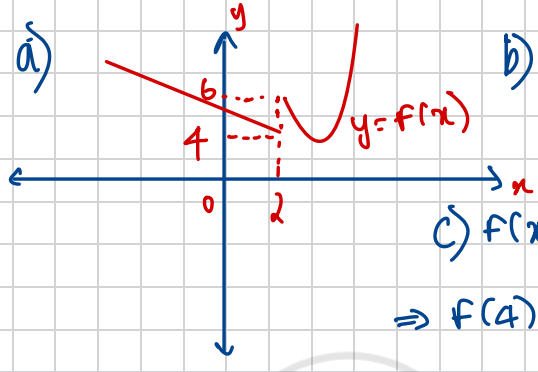
i)



ii) Range: $f(x) \geq 0$ iii) one-to-one

④ $f(x) = \begin{cases} 5 - \frac{1}{2}x, & x \in R, \ x < 2 \\ (x-4)^2 + 2, & x \in R, \ x \geq 2 \end{cases}$

a) $y = f(x)$ a)



b) $f(x)$ is a many-to-one mapping for all $x \in R$, so it is a function. $f(2) = 6$

c) $f(x) = (x-4)^2 + 2 \Rightarrow x = 4$ at min

$\Rightarrow f(4) = 2 \Rightarrow$ Range: $f(x) \geq 2$

d) $f(x) = 11$; $x < 2 \Rightarrow 5 - \frac{1}{2}x = 11 \Rightarrow -\frac{1}{2}x = 6$ $\boxed{x = -12}$

$f(x) = 11$, $x \geq 2 \Rightarrow (x-4)^2 + 2 = 11 \Rightarrow (x-4)^2 = 9 \Rightarrow x - 4 = \pm 3$

$\Rightarrow x - 4 = 3 \Rightarrow \boxed{x = 7}$

⑤ $f: x \to x^2 - 3x + 4$, $x \in R$, $0 \leq x < 4$

a) $x = -\frac{b}{2a} \Rightarrow x = -\frac{(-3)}{2(1)} = \frac{+3}{2}$

$\Rightarrow F\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 4 = \frac{7}{4}$ $\Rightarrow F(0) = 0^2 - 3(0) + 4 = 4$

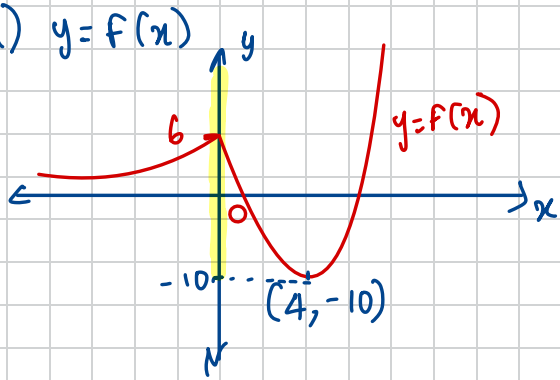
$\Rightarrow F(4) = 4^2 - 3(4) + 4 = 8$

Range: $\frac{7}{4} \leq f(x) < 8$

b) many-to-one

⑥ $f(x) = \begin{cases} e^x + 5, & x \in R, \ x \leq 0 \\ x^2 - 8x + 6, & x \in R, \ x \geq 0 \end{cases}$

a) $y = f(x)$



b) $x^2 - 8x + 6 \Rightarrow (x-4)^2 - 4^2 + 6$

$\Rightarrow (x-4)^2 - 16 + 16 = (x-4)^2 - 10$

Range: $f(x) \geq -10$

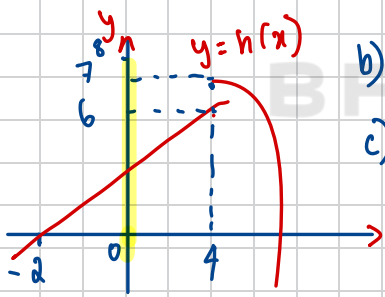
(7) $g(x) = x^2 - 5x + 8$, $x \in R$, $x \geq a$

$\Rightarrow x = -\frac{b}{2a} \Rightarrow -\frac{(-5)}{2(1)} = \frac{5}{2}$ $\boxed{a: \frac{5}{2}}$, one-to-one function

$\Rightarrow (x - 2\cdot 5)^2 - 2\cdot 5^2 + 8 \Rightarrow (x - 2\cdot 5)^2 + \frac{7}{4}$ $x = \frac{5}{2} = a$

(8) $h(x): \begin{cases} 2+x, & x \in R, \ x \leq 4 \\ 10x - x^2 - 17, & x \in R, \ x > 4 \end{cases}$

a)



b) $h(4) = 2 + 4 = 6$ $(x \leq 4)$

c) $h(x)$: Range: $f(x) \leq 8$

d) $h(x) = 5$, $x \leq 4$

$\Rightarrow 2 + x = 5 \Rightarrow \boxed{x = 3}$

ii) $h(x) = 5$, $x > 4 \Rightarrow 10x - x^2 - 17 = 5 \Rightarrow x^2 - 10x + 17 = -5$

$\Rightarrow (x-5)^2 - 8 = -5 \Rightarrow x - 5 = \pm\sqrt{3}$ $x = 5 \pm \sqrt{3}$

$x = 5 + \sqrt{3}$, $x > 4$