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14.6

1) a) $6^x = 31$
(base law)
$\hookrightarrow \log_6(31) = x$
$\underline{x = 1.92}$.

b) $3^x = 70$
(base law)
$\hookrightarrow \log_3(70) = x$
$\underline{x = 3.87}$.

c) $5^{2x} = 640$
(base law)
$\hookrightarrow \log_5(640) = 2x$
$\frac{ANS}{2} = \frac{\cancel{2}x}{\cancel{2}}$
$\underline{x = 2.01}$

2) a) $5^{x+3} = 0.3$
(base law)
$\hookrightarrow \log_5(0.3) = x+3$
$ANS - 3 = x$
$\underline{x = -3.75}$.

b) $4^{2x-5} = 85$
$\log_4(85) = 2x-5$
$\frac{ANS+5}{2} = \frac{\cancel{2}x}{\cancel{2}}$
$\underline{x = 4.10}$.

c) $7^{1-4x} = 11$
(base law)
$\hookrightarrow \log_7(11) = 1-4x$
$\frac{ANS-1}{-4} = \frac{-\cancel{4}x}{\cancel{-4}}$
$\underline{x = -0.0581}$.

3) a) $5^{2x} - 11(5^x) + 30 = 0$
Let $y = 5^x$
$y^2 - 11y + 30 = 0$
$y = 6$ or $y = 5$
$5^x = 6$ or $5^x = 5$
(base law)
$\hookrightarrow \log_5(6) = x$ $\quad \log_5(5) = x$
$\underline{x = 1.11}$ $\quad \underline{x = 1}$.

b) $4^{2x} - 19(4^x) + 60 = 0$
Let $y = 4^x$
$y^2 - 19y + 60 = 0$
$y = 15$ or $y = 4$
$4^x = 15$ or $4^x = 4$
(base law)
$\hookrightarrow \log_4(15) = x$ $\quad \log_4(4) = x$
$\underline{x = 1.95}$ $\quad \underline{x = 1}$.

c) $5^{2x+1} - 19(5^x) + 18 = 0$
(break down the exponents)
$5^1(5^{2x}) - 19(5^x) + 18 = 0$
Let $y = 5^x$
$5y^2 - 19y + 18 = 0$
$y = 2$ or $y = \frac{9}{5}$
$5^x = 2$ or $5^x = \frac{9}{5}$
(base law)
$\hookrightarrow \log_5(2) = x$ $\quad \log_5(\frac{9}{5}) = x$
$\underline{x = 0.431}$. $\quad \underline{x = 0.365}$.

④ a) $9u^2 - 3u - 20 = 0$

$(3u-5)(3u+4)$

$u = \frac{5}{3}$ or $u = -\frac{4}{3}$

b) $3^{2x+2} - 3(3^x) - 20 = 0$

(separate the exponents)

$3^2(3^{2x}) - 3(3^x) - 20 = 0$

$9(3^{2x}) - 3(3^x) - 20 = 0$

Let $y = 3^x$

$9u^2 - 3u - 20 = 0$

c) $9u^2 - 3u - 20 = 0$

$u = \frac{5}{3}$ or $u = -\frac{4}{3}$

$3^x = \frac{5}{3}$ or $3^x = -\frac{4}{3}$

(base law)

$\hookrightarrow \log_3(5/3) = x$ $\quad \cancel{\log_3(-4/3)}$

(ERROR)

$x = 0.465$

* No real solution as you can't take the logarithm of a negative number

⑤ a) $8^x = 5^{x+3}$

(take logarithm on both sides)

$\log(8^x) = \log(5^{x+3})$

(power law)

$\hookrightarrow x\log(8) = (x+3)\log(5)$ expand brackets

$x\log(8) = x\log(5) + 3\log(5)$

$x\log(8) - x\log(5) = 3\log(5)$

$x(\log(8) - \log(5)) = 3\log(5)$

$$x = \frac{3\log(5)}{\log(8) - \log(5)}$$

$x = 10.3$

(5) b) $7^{3-x} = 3^{2x}$

(take logarithm on both sides)

$\log(7^{3-x}) = \log(3^{2x})$

(Power law)

$\hookrightarrow (3-x)\log(7) = 2x\log(3)$

expand brackets

$3\log(7) - x\log(7) = 2x\log(3)$

$3\log(7) = 2x\log(3) + x\log(7)$

$3\log(7) = x(2\log(3) + \log(7))$

$$x = \frac{3\log(7)}{2\log(3) + \log(7)}$$

$\underline{x = 1.41}$.

c) $6^{2x} = 4^{x+3}$

(take logarithm on both sides)

$\log(6^{2x}) = \log(4^{x+3})$

(power law)

$\hookrightarrow 2x\log(6) = (x+3)\log(4)$ expand brackets

$2x\log(6) = x\log(4) + 3\log(4)$

$2x\log(6) - x\log(4) = 3\log(4)$

$x(2\log(6) - \log(4)) = 3\log(4)$

$$x = \frac{3\log(4)}{2\log(6) - \log(4)}$$

$x = \underline{1.89}$.

(6) a) $7^{2x-5} = 400$

(take logarithm on both sides)

$\log(7^{2x-5}) = \log(400)$

(Power law)

$\hookrightarrow (2x-5)\log(7) = \log(400)$

expand brackets

$2x\log(7) - 5\log(7) = \log(400)$

$2x\log(7) = \log(400) + 5\log(7)$

$$\frac{x(\cancel{2\log(7)})}{\cancel{2\log(7)}} = \frac{\log(400) + 5\log(7)}{2\log(7)}$$

$\underline{x = 4.04}$.

⑥ b) $6^{2x} + 4(6^x) - 21 = 0$

Let $y = 6^x$

$y^2 + 4y - 21 = 0$

$y = 3$ or $y = -7$

$6^x = 3$ or $6^x = -7$

(base law)

$\hookrightarrow \log_6(3) = x$

$\underline{x = 0.613}$.

~~$\log_6(-7) = x$~~

ERROR as you can't take the logarithm of a negative number.

⑦ a) $4^{x+2} = 5^{x-1}$

(take logarithm of both sides)

$\log(4^{x+2}) = \log(5^{x-1})$

(Power law)

$\hookrightarrow (x+2)\log(4) = (x-1)\log(5)$

expand brackets

$x\log(4) + 2\log(4) = x\log(5) - \log(5)$

$x\log(4) - x\log(5) = -2\log(4) - \log(5)$

$$\frac{x\,(\cancel{\log(4) - \log(5)})}{\cancel{\log(4) - \log(5)}} = \frac{-2\log(4) - \log(5)}{\log(4) - \log(5)}$$

$\underline{x = 19.6}$.

b) $\log_7(5-x) = 0.8$

(base law)

$\hookrightarrow 7^{0.8} = 5 - x$

$x = 5 - 7^{0.8}$

$x = \underline{0.257}$.

⑧ a) $y = -2^x + 7$

Find y-intercept:

$\hookrightarrow y = -2^0 + 7$

$y = 6 \quad (0,6)$

Find the root:

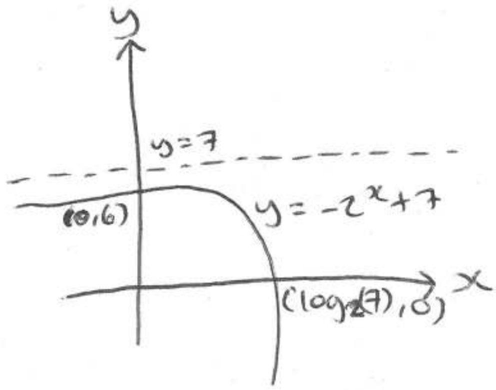
$\hookrightarrow 0 = -2^x + 7$

$2^x = 7$

(base law)

$\hookrightarrow \log_2(7) = x$

Asymptote is $y = 7$ as the original function $y = -2^x$ has an asymptote at $y = 0$. So, the graph is shifted upwards by 7 units



(8) b) $-2^x + 7 = -6$

$7 + 6 = 2^x$

$2^x = 13$

(base law)

$\hookrightarrow \log_2(13) = x$

$\underline{x = 3.70}$.

(9) a) $9^x - 3^{x+1} - 10 = 0$

(Break down the exponents)

$\hookrightarrow (3^2)^x - 3^1(3^x) - 10 = 0$

$3^{2x} - 3(3^x) - 10 = 0$

Let $u = 3^x$

$\underline{u^2 - 3u - 10 = 0}$.

b) $u^2 - 3u - 10 = 0$

$u = 5$ or $u = -2$

$3^x = 5$ or $3^x = -2$

(base law)

$\hookrightarrow \log_3(5) = x$

$\underline{x = 1.46}$.

$\cancel{\log_3(-2)} = x$

ERROR as logarithm can't have a negative solution