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13.3 finding functions

1) a) $\int (4x^3 - 10x^2)\,dx$

$\frac{4x^4}{4} - \frac{10x^3}{3} + c$

$x^4 - \frac{10}{3}x^3 + c$

$f(3) = 6$

$(3)^4 - \frac{10}{3}(3)^3 + C = 6$

$-9 + C = 6$

$C = 15$

$y = x^4 - \frac{10}{3}x^3 + 15$

b) $\int \left(\frac{5}{\sqrt{x}} + \frac{1}{2}x^2\right) dx$

$\frac{5x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{\frac{1}{2}x^3}{3} + c$

$10x^{\frac{1}{2}} + \frac{1}{6}x^3 + c$

$f(16) = 65$

$65 = 10(16)^{\frac{1}{2}} + \frac{1}{6}(16)^3 + C$

$65 = \frac{2168}{3} + C$

$C = -\frac{1973}{3}$

$y = 10x^{\frac{1}{2}} + \frac{1}{6}x^3 - \frac{1973}{3}$

2) a) $\int \left(\frac{3 - x^2}{x^4}\right) dx$

$\frac{3}{x^4} - \frac{x^2}{x^4} + c$

$3x^{-4} - x^2x^{-4} + c$

$\frac{3x^{-3}}{-3} - \frac{x^{-1}}{-1} + c$

$-x^{-3} + x^{-1} + c$

$f(-1) = 4$

$-(-1)^{-3} + (-1)^{-1} + C = 4$

$0 + C = 4$

$C = 4$

$y = -x^{-3} + x^{-1} + 4$

b) $\int (x - 5)^2\,dx$

$x^2 - 10x + 25 + c$

$\frac{x^3}{3} - \frac{10x^2}{2} + 25x + c$

$\frac{x^3}{3} - 5x^2 + 25x + c$

$f(2) = 18$

$\frac{2^3}{3} - 5(2)^2 + 25(2) + C = 18$

$\frac{98}{3} + C = 18$

$C = -\frac{44}{3}$

$y = \frac{x^3}{3} - 5x^2 + 25x - \frac{44}{3}$

3) $\int (8x^{-\frac{1}{2}} - 4x^{\frac{1}{2}})\,dx$

$\frac{8x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$16x^{\frac{1}{2}} - \frac{8}{3}x^{\frac{3}{2}} + c$

$f(9) = -30$

$16(9)^{\frac{1}{2}} - \frac{8}{3}(9)^{\frac{3}{2}} + c = -30$

$-24 + C = -30$

$C = -6$

$y = 16x^{\frac{1}{2}} - \frac{8}{3}x^{\frac{3}{2}} - 6$

4) $\int (10 - 6x - 4x^2)\,dx$

$10x - \frac{6x^2}{2} - \frac{4x^3}{3} + c$

$10x - 3x^2 - \frac{4}{3}x^3 + c$

$f(3) = -15$

$10(3) - 3(3)^2 - \frac{4}{3}(3)^3 + c = -15$

$-33 + c = -15$

$c = 18$

$y = 10x - 3x^2 - \frac{4}{3}x^3 + 18$

5) $\int \left(\frac{5\sqrt{x}}{x^2} - \frac{3x^{\frac{3}{2}}}{x^2}\right) dx$

$5x^{\frac{1}{2}} \cdot x^{-2} - 3x^{\frac{3}{2}} \cdot x^{-2} + c$

$\frac{5x^{-\frac{1}{2}}}{-\frac{1}{2}} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$-10x^{-\frac{1}{2}} - 6x^{\frac{1}{2}} + c$

$f(4) = 7$

$7 = -10(4)^{-\frac{1}{2}} - 6(4)^{\frac{1}{2}} + c$

$-17 + c = 7$

$c = 24$

$y = -10x^{-\frac{1}{2}} - 6x^{\frac{1}{2}} + 24$

6) $\int (6x - 4x^{-\frac{1}{3}})\,dx$

$\frac{6x^2}{2} - \frac{4x^{\frac{2}{3}}}{\frac{2}{3}} + c$

$3x^2 - 6x^{\frac{2}{3}} + c$

$f(8) = 112$

$3(8)^2 - 6(8)^{\frac{2}{3}} + c = 112$

$168 + c = 112$

$c = -56$

$y = 3x^2 - 6x^{\frac{2}{3}} - 56$

7) a) $\int \left(\frac{(1-5x)^3}{\sqrt{x^3}}\right) dx$

	1	3	3	1
1	1	1	1	1
$-5x$	1	$-5x$	$(-5x)^2$	$(-5x)^3$

$p'(x) = \frac{1 - 15x + 75x^2 - 125x^3}{x^{\frac{3}{2}}} =$

$= x^{-\frac{3}{2}} - 15x^{-\frac{1}{2}} + 75x^{\frac{1}{2}} - 125x^{\frac{3}{2}}$

b) $\int \left(x^{-\frac{3}{2}} - 15x^{-\frac{1}{2}} + 75x^{\frac{1}{2}} - 125x^{\frac{3}{2}}\right)$

$\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - \frac{15x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{75^{\frac{3}{2}}}{\frac{3}{2}} - \frac{125x^{\frac{5}{2}}}{\frac{5}{2}} + C$

$-2x^{-\frac{1}{2}} - 30x^{\frac{1}{2}} + 50x^{\frac{3}{2}} - 50x^{\frac{5}{2}} + C$

$p'(1) = -24$

$-24 = -2(1)^{-\frac{1}{2}} - 30(1)^{\frac{1}{2}} + 50(1)^{\frac{3}{2}} - 50(1)^{\frac{5}{2}} + C$

$-24 = -32 + C \qquad C = 8$

$y = -2x^{-\frac{1}{2}} - 30x^{\frac{1}{2}} + 50x^{\frac{3}{2}} - 50x^{\frac{5}{2}} + 8$

8) $\int \left(\frac{3}{2}x^2 - \frac{1}{4x^{1/2}}\right) dx$

$\frac{\frac{3}{2}x^3}{3} - \frac{1}{4}x^{-\frac{1}{2}} + c$

$\frac{1}{2}x^3 - \frac{\frac{1}{4}x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$\frac{1}{2}x^3 - \frac{1}{2}x^{\frac{1}{2}} + c$

$h(4) = -10$

$-10 = \frac{1}{2}(4)^3 - \frac{1}{2}(4)^{\frac{1}{2}} + c$

$-10 = 31 + c$

$c = -41$

$h(2) = \frac{1}{2}(2)^3 - \frac{1}{2}(2)^{\frac{1}{2}} - 41$

$h(2) = -37 - \frac{1}{2}\sqrt{2}$