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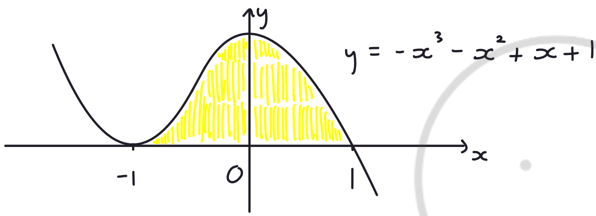
13.5 Areas under curves

1.



$$\int_{-1}^{1} (-x^3 - x^2 + x + 1)\,dx$$

$$\left[\frac{-x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} + x\right]_{-1}^{1}$$

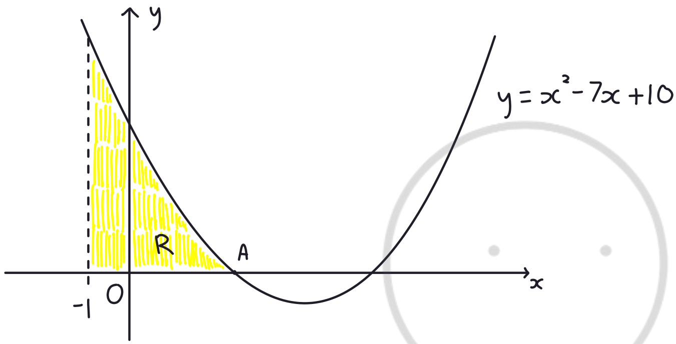
Substitute $x=1$ and $x=-1$

$$\left(\frac{-1}{4} - \frac{1}{3} + \frac{1}{2} + 1\right) - \left(-\frac{1}{4} + \frac{1}{3} + \frac{1}{2} - 1\right)$$

$$\left(\frac{11}{12}\right) - \left(-\frac{5}{12}\right)$$

Area $= \dfrac{4}{3}$

2.



a) Solve $x^2-7x+10=0$

$x=5$ or $x=2$

x-coordinate is closer to 0

so A is $(2,0)$

b) $\int_{-1}^{2} (x^2-7x+10)\,dx$

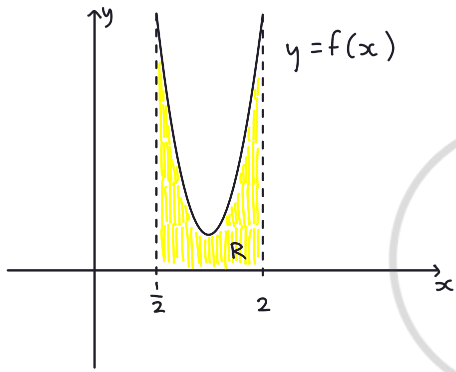
$\left[\frac{x^3}{3}-\frac{7x^2}{2}+10x\right]_{-1}^{2}$

Substitute $x=2$ and $x=-1$

$\left(\frac{8}{3}-14+20\right)-\left(-\frac{1}{3}-\frac{7}{2}-10\right)$

$R=\frac{45}{2}$

3.



$$f(x) = 5x^2 + \frac{2}{x^3} - 5$$

$$= 5x^2 + 2x^{-3} - 5$$

$$\int_{\frac{1}{2}}^{2} \left(5x^2 + 2x^{-3} - 5\right) dx$$

$$\left[\frac{5x^3}{3} + \frac{2x^{-2}}{-2} - 5x\right]_{\frac{1}{2}}^{2}$$

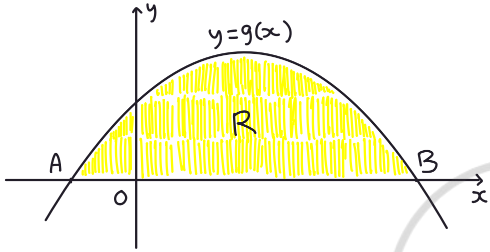
$$\left[\frac{5x^3}{3} - x^{-2} - 5x\right]_{\frac{1}{2}}^{2}$$

Substitute $x = 2$ and $x = \frac{1}{2}$

$$\left(\frac{40}{3} - \frac{1}{4} - 10\right) - \left(\frac{5}{24} - 4 - \frac{5}{2}\right)$$

$$R = \frac{75}{8}$$

4.



a) $g(x) = 12 + 4x - x^2$

Solve: $-x^2 + 4x + 12$

$x = 6$ or $x = -2$

so A is $(-2, 0)$ and B is $(6, 0)$

b) $$\int_{-2}^{6} (-x^2 + 4x + 12)\, dx$$

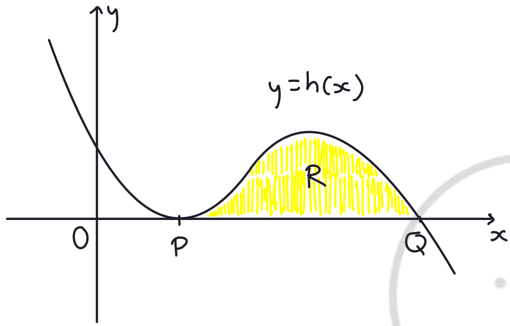
$$\left[\frac{-x^3}{3} + \frac{4x^2}{2} + 12x \right]_{-2}^{6}$$

$$= \left[\frac{-x^3}{3} + 2x^2 + 12x \right]_{-2}^{6}$$

$$(-72 + 72 + 72) - \left(\frac{8}{3} + 8 - 24 \right)$$

$$R = \frac{256}{3}$$

5.



$h(x) = -(x-1)^2(x-4)$

$x = 1 \quad x = 4$

so P is $(1,0)$ and Q is $(4,0)$

$$\begin{aligned}
&-(x-1)^2(x-4) \\
&= -(x^2-2x+1)(x-4) \\
&= -(x^3-4x^2-2x^2+8x+x-4) \\
&= -(x^3-6x^2+9x-4) \\
&= -x^3+6x^2-9x+4
\end{aligned}$$

$$\int_1^4 (-x^3+6x^2-9x+4)\,dx$$

$$\left[\frac{-x^4}{4}+\frac{6x^3}{3}-\frac{9x^2}{2}+4x\right]_1^4$$

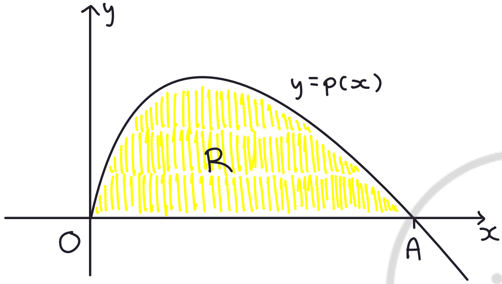
$$\left[\frac{-x^4}{4}+2x^3-\frac{9x^2}{2}+4x\right]_1^4$$

Substitute $x=4$ and $x=1$

$$(-64+128-72+16)-\left(-\frac{1}{4}+2-\frac{9}{2}+4\right)$$

$$R = \frac{27}{4}$$

6.



a) $p(x) = 8\sqrt{x} - x^2$

$= 8x^{\frac{1}{2}} - x^2$

$x^2 = 8x^{\frac{1}{2}}$

divide by $x^{\frac{1}{2}}$ on both sides

$x^{\frac{3}{2}} = 8$

$x = \sqrt[\frac{3}{2}]{8}$

$x = 4$

$A = (4, 0)$

b) $\displaystyle\int_0^4 (8\sqrt{x} - x^2)\,dx$

$= \displaystyle\int_0^4 (8x^{\frac{1}{2}} - x^2)\,dx$

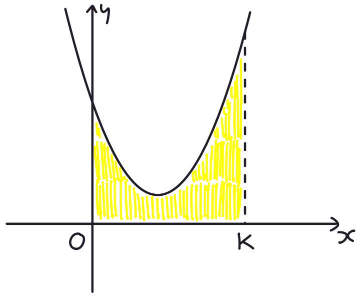
$\left[\dfrac{8x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{x^3}{3}\right]_0^4 = \left[\dfrac{16x^{\frac{3}{2}}}{3} - \dfrac{x^3}{3}\right]_0^4$

$\left(\dfrac{128}{3} - \dfrac{64}{3}\right)$

$R = \dfrac{64}{3}$

7.

$$y = 6x^2 - 12x + 9$$



$$\int_0^k (6x^2 - 12x + 9)\, dx$$

$$\left[\frac{6x^3}{3} - \frac{12x^2}{2} + 9x\right]_0^k$$

$$\left[2x^3 - 6x^2 + 9x\right]_0^k$$

Substitute $x = k$ and $x = 0$

$$(2k^3 - 6k^2 + 9k)$$

$$27 = 2k^3 - 6k^2 + 9k$$
$$2k^3 - 6k^2 + 9k - 27 = 0$$

Solve for k:
$k = 3$