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12.11 Modelling with differentiation

1. $A = 6x^2$

$$\frac{dA}{dx} = 12x$$

2. $V = \dfrac{20000}{t^2}$

a) $V = 20000t^{2(-1)} = 20000t^{-2}$

$$\frac{dV}{dt} = -40000t^{-3}$$

b) when $t = 10$

$$\frac{dV}{dt} = -40000(10)^{-3}$$

$$\frac{dV}{dt} = -40$$

3.

a) $v = (8 - \sqrt{t})\, ms^{-1}$

$$8 - \sqrt{t} = 0$$
$$-\sqrt{t} = -8$$
$$\sqrt{t} = 8$$
$$t = \pm 64$$

t can't be negative

$\therefore t = 64$ seconds

b) $v = 8 - t^{\frac{1}{2}}$

$$\frac{dv}{dt} = -t^{\frac{1}{2} - 1} \times \frac{1}{2}$$

$$\frac{dv}{dt} = -\frac{1}{2} t^{-\frac{1}{2}}$$

c) $a = \frac{dv}{dt}$

when $t = 9$

$$a = -\frac{1}{2}(9)^{-\frac{1}{2}}$$

$$a = -\frac{1}{6}\, ms^{-2}$$

4. $V = \frac{4}{3}\pi r^3$

a) $\frac{dV}{dr} = \frac{4}{3} \times 3\pi r^{3-1}$

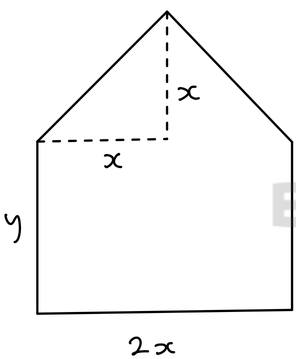
$\frac{dV}{dr} = 4\pi r^2$

b) When $r = 4$

$\frac{dV}{dr} = 4\pi(4)^2$

$\frac{dV}{dr} = 64\pi \text{ cm}^3 \text{ per cm}$

5.



a) $\sqrt{x^2 + x^2} = \sqrt{2x}$

$80 = 2y + 2x + 2\sqrt{2x}$

$-2y = -80 + 2x + 2\sqrt{2x}$

$2y = 80 - 2x - 2\sqrt{2x}$

$y = 40 - x - \sqrt{2x}$

$y = 40 - x(1 + \sqrt{2})$

b) $A = 2xy + x^2$

$= 2x(40 - x(1+\sqrt{2})) + x^2$

$= 2x(40 - x - \sqrt{2x}) + x^2$

$= 80x - 2x^2 - 2x^2\sqrt{2} + x^2$

$= 80x - x^2 - 2x^2\sqrt{2}$

$= 80x - x^2(1 + 2\sqrt{2})$

c) $A = 80x - x^2(1+2\sqrt{2})$

$A' = 80 - 2x(1 + 2\sqrt{2})$

$A' = 0$

$80 - 2x(1+2\sqrt{2}) = 0$

$80 = 2x(1+2\sqrt{2})$

$$x = \frac{80}{2(1+2\sqrt{2})}$$

When $x = \dfrac{80}{2(1+2\sqrt{2})}$

max. value of $A = 80\left(\dfrac{80}{2(1+2\sqrt{2})}\right) - \left(\dfrac{80}{2(1+2\sqrt{2})}\right)^2(1+2\sqrt{2})$

exact max. value of $A = \dfrac{1600(2\sqrt{2}-1)}{7}$ m^2

6.

a) $V = \pi r^2 h$

$1024 = \pi r^2 h$

$r = \sqrt{\dfrac{1024}{h}}$

Substitute $r = \sqrt{\dfrac{1024}{h}}$ into $SA = 2\pi r^2 + 2\pi r h$

$$2\pi\left(\frac{1024}{h}\right) + 2\pi\sqrt{\frac{1024}{h}}\,h$$

$$= \frac{2048\pi}{h} + 2\pi\left(\frac{32}{\sqrt{h}}\right)h$$

$$= \frac{2048\pi}{h} + 64\pi\sqrt{h}$$

b) $SA = \dfrac{2048\pi}{h} + 64\pi\sqrt{h}$

$$= 2048\pi h^{-1} + 64\pi h^{\frac{1}{2}}$$

$$\frac{dS}{dh} = -2048\pi h^{-2} + 32\pi h^{-\frac{1}{2}}$$

$$= -\frac{2048}{h^2} + \frac{32\pi}{\sqrt{h}}$$

$$\frac{2048}{h^2} = \frac{32}{\sqrt{h}}$$

$$2048\sqrt{h} = 32h^2$$

$$64\sqrt{h} = h^2$$

$$64^2 = h^3$$

$$4096 = h^3$$

$$h = 16$$

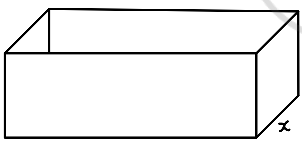
substitute $h=16$ into $\dfrac{2048\pi}{h} + 64\pi\sqrt{h}$

$$\frac{2048\pi}{16} + 64\pi\sqrt{16}$$

max. value of surface area $= 384\pi \text{ cm}^2$

7.

a)



let length be y

Total area $= x^2 + x^2 + xy + xy + xy$

$= 2x^2 + 3xy$

$$2x^2 + 3xy = 608$$

$$3xy = 608 - 2x^2$$

$$y = \frac{608 - 2x^2}{3x}$$

$$V = x^2 y$$

$$V = x^2\left(\frac{608 - 2x^2}{3}\right)$$

$$V = \frac{608x^2 - 2x^4}{3x}$$

$$V = \frac{608x - 2x^3}{3}$$

b) $V = \frac{608x - 2x^3}{3} = \frac{608x}{3} - \frac{2x^3}{3}$

$$\frac{dV}{dx} = \frac{608}{3} - 2x^2$$

$$\frac{dV}{dx} = 0$$

$$\frac{608}{3} - 2x^2 = 0$$

$$\frac{608}{3} = 2x^2$$

$$\frac{304}{3} = x^2$$

$$x = \sqrt{\frac{304}{3}}$$

Substitute x into volume equation

$$V = \frac{608\left(\sqrt{\frac{304}{3}}\right) - 2\left(\sqrt{\frac{304}{3}}\right)^3}{3}$$

max. value of $V = 1360.1 \text{ cm}^3$ (1 dp)

c) Find $\dfrac{d^2V}{dx^2} = -4x$

$x > 0$ so $\dfrac{d^2V}{dx^2} < 0$ so V is a maximum

8.

a) $V_{cylinder} = \pi r^2 h$

$V_{hemisphere} = \dfrac{2}{3}\pi r^3$

$$V = V_{cylinder} + V_{hemisphere}$$
$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$A_{cylinder} = 2\pi rh + 2\pi r^2$ (excluding base)

$A_{hemisphere} = 2\pi r^2$

$$A = A_{cylinder} + A_{hemisphere}$$
$$= 2\pi rh + \pi r^2 + 2\pi r^2$$
$$= 2\pi rh + 3\pi r^2 = 800\pi$$

$$2\pi rh + 3\pi r^2 = 800\pi$$
$$2rh + 3r^2 = 80$$

$$2rh = 800 - 3r^2$$
$$h = \frac{800 - 3r^2}{2r}$$

Substitute h into Volume equation

$$V = \pi r^2 \left(\frac{800 - 3r^2}{2r} \right) + \frac{2}{3} \pi r^3$$

$$V = \frac{\pi r (800 - 3r^2)}{2} + \frac{2}{3} \pi r^3$$

$$= \frac{800\pi r - 3\pi r^3}{2} + \frac{2\pi r^3}{3}$$

$$= \frac{2400\pi r - 9\pi r^3 + 4\pi r^3}{6} = \frac{2400\pi r - 5\pi r^3}{6}$$

$$= \frac{\pi r (2400 - 5r^2)}{6}$$

b) $$V = \frac{\pi r (2400 - 5r^2)}{6} = \frac{2400\pi r - 5\pi r^3}{6}$$

$$\frac{dV}{dr} = \frac{2400\pi - 15\pi r^2}{6}$$

$$\frac{dV}{dr} = 0$$

$$\frac{2400\pi - 15\pi r^2}{6} = 0$$

$$2400\pi - 15\pi r^2 = 0$$
$$15\pi r^2 = 2400\pi$$
$$15 r^2 = 2400$$
$$r^2 = 160$$
$$r = \sqrt{160} = 4\sqrt{10}$$

Substitute $r = 4\sqrt{10}$ into volume equation

$$V = \frac{\pi\left(4\sqrt{10}\right)\left(2400 - 5(160)\right)}{6}$$

max. value of $V = 10600 \text{ cm}^3$ (3 sf)