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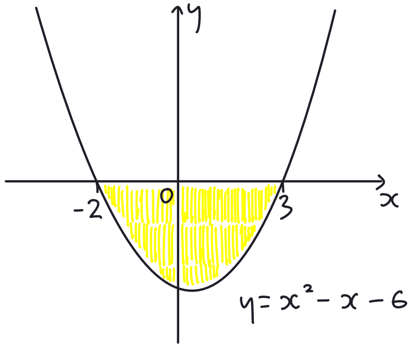
13.6 Areas under the x-axis

1.



$$\int_{-2}^{3} (x^2 - x - 6)\, dx$$

$$\left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_{-2}^{3}$$

Substitute $x = 3$ and $x = -2$

$$\left(9 - \frac{9}{2} - 18\right) - \left(-\frac{8}{3} - 2 + 12\right)$$

$$= -\frac{125}{6}$$

Area $= \frac{125}{6}$

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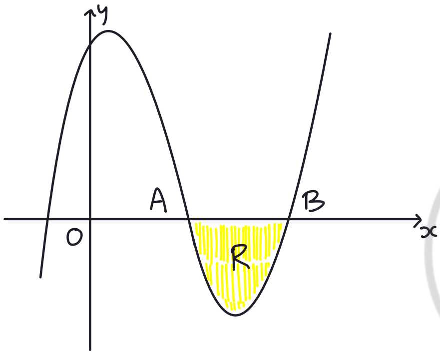
2.



$y=(x+1)(x-2)(x-4)$

a) $y=(x+1)(x-2)(x-4)$

$x=-1,\ x=2,\ x=4$

$A=2,\quad B=4$

b) $(x+1)(x-2)(x-4)$

$(x^2-x-2)(x-4)$

$x^3-4x^2-x^2+4x-2x+8$

$= x^3-5x^2+2x+8$

$$\int_2^4 \left(x^3-5x^2+2x+8\right)dx$$

$$\left[\frac{x^4}{4}-\frac{5x^3}{3}+\frac{2x^2}{2}+8x\right]_2^4 = \left[\frac{x^4}{4}-\frac{5x^3}{3}+x^2+8x\right]_2^4$$

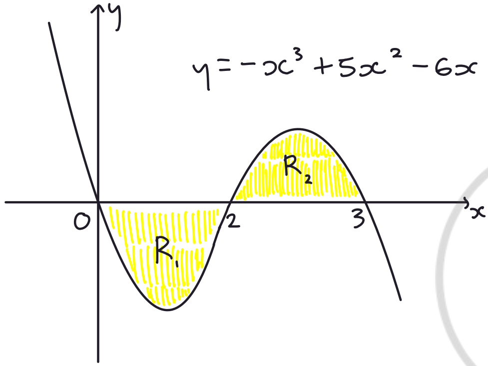
Substitute $x=4$ and $x=2$

$$\left(64-\frac{320}{3}+16+32\right)-\left(4-\frac{40}{3}+4+16\right) = \frac{-16}{3}$$

$R = \dfrac{16}{3}$

BF MATHS

3.



a) R_1 :

$$\int_0^2 (-x^3 + 5x^2 - 6x)\,dx$$

$$\left[\frac{-x^4}{4} + \frac{5x^3}{3} - \frac{6x^2}{2}\right]_0^2 = \left[\frac{-x^4}{4} + \frac{5x^3}{3} - 3x^2\right]_0^2$$

Substitute $x = 2$ and $x = 0$

$$\left(-4 + \frac{40}{3} - 12\right)$$

$$= -\frac{8}{3}$$

$$R_1 = \frac{8}{3}$$

BF MATHS

b) R_2:

$$\int_2^3 (-x^3 + 5x^2 - 6x)\,dx$$

$$\left[\frac{-x^4}{4} + \frac{5x^3}{3} - \frac{6x^2}{2}\right]_2^3 = \left[\frac{-x^4}{4} + \frac{5x^3}{3} - 3x^2\right]_2^3$$

Substitute $x = 3$ and $x = 2$

$$\left(-\frac{81}{4} + 45 - 27\right) - \left(-4 + \frac{40}{3} - 12\right)$$

$$R_2 = \frac{5}{12}$$

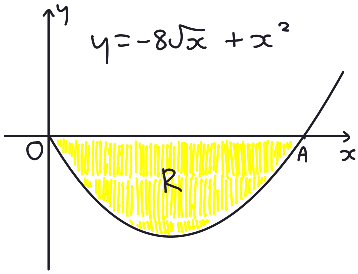
BF MATHS

c) Total area $= R_1 + R_2$

$$= \frac{8}{3} + \frac{5}{12}$$

$$= \frac{37}{12}$$

4.



a) $y = -8\sqrt{x} + x^2$

$= -8x^{\frac{1}{2}} + x^2$

$8x^{\frac{1}{2}} = x^2$

$8 = x^{\frac{3}{2}}$

$x = \sqrt[\frac{3}{2}]{8}$

$x = 4$

so A is $(4,0)$

b) $\displaystyle\int_0^4 \left(-8x^{\frac{1}{2}} + x^2\right)dx$

$\left[\dfrac{-8x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{x^3}{3}\right]_0^4 = \left[-\dfrac{16}{3}x^{\frac{3}{2}} + \dfrac{x^3}{3}\right]_0^4$

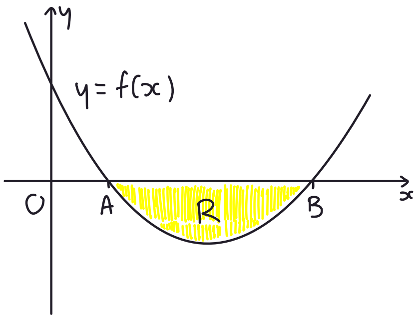
Substitute $x=4$ and $x=0$

$\left(-\dfrac{128}{3} + \dfrac{64}{3}\right) = -\dfrac{64}{3}$

$R = \dfrac{64}{3}$

BF MATHS

5.



a) $f(x) = \frac{1}{2}x^2 - \frac{13}{2}x + 15$

multiply by 2 on both sides:

$x^2 - 13x + 30$
$(x-10)(x-3)$
$x = 10 \quad x = 3$

A is $(3,0)$ and B is $(10,0)$

b) $\displaystyle\int_3^{10} \left(\frac{1}{2}x^2 - \frac{13}{2}x + 15\right) dx$

$$\left[\frac{\frac{1}{2}x^3}{3} - \frac{\frac{13}{2}x^2}{2} + 15x\right]_3^{10} \quad \left[\frac{1}{6}x^3 - \frac{13}{4}x^2 + 15x\right]_3^{10}$$

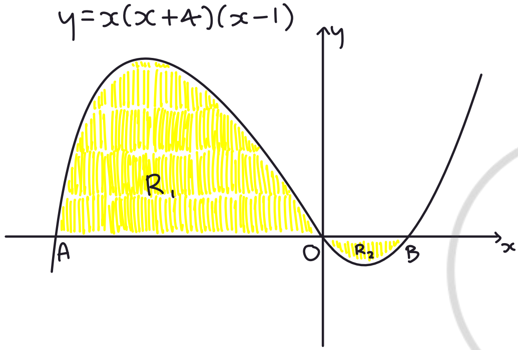
Substitute $x = 10$ and $x = 3$

$$\left(\frac{500}{3} - 325 + 150\right) - \left(\frac{9}{2} - \frac{117}{4} + 45\right) = -\frac{343}{12}$$

$R = \frac{343}{12}$

BF MATHS

6.



a) $y = x(x+4)(x-1)$

$x = 0, \quad x = -4, \quad x = 1$

A is $(-4, 0)$

B is $(1, 0)$

BF MATHS

b) $x(x+4)(x-1)$

$x(x^2+3x-4)$

x^3+3x^2-4x

R_1 :

$$\int_{-4}^{0} (x^3+3x^2-4x)\,dx$$

$$\left[\frac{x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2}\right]_{-4}^{0} = \left[\frac{x^4}{4} + x^3 - 2x^2\right]_{-4}^{0}$$

Substitute $x=0$ and $x=-4$

$$\left(64 - 64 - 32\right) = -32$$

$R_1 = 32$

R_2:

$$\int_0^1 (x^3 + 3x^2 - 4x)\,dx$$

$$\left[\frac{x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2}\right]_0^1 = \left[\frac{x^4}{4} + x^3 - 2x^2\right]_0^1$$

Substitute $x=1$ and $x=0$

$$\left(\frac{1}{4} + 1 - 2\right) = -\frac{3}{4}$$

$$R_2 = \frac{3}{4}$$

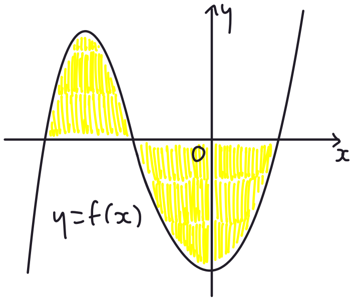
Total area $= R_1 + R_2$

$$= 32 + \frac{3}{4}$$

$$= \frac{131}{4}$$

BF MATHS

7.



a) Using factor theorem:

$$f(2) = 2^3 + 6(2)^2 - 4(2) - 24$$
$$= 0$$

b) $f(2) = 0$

so $(x-2)$ is a factor of $f(x)$

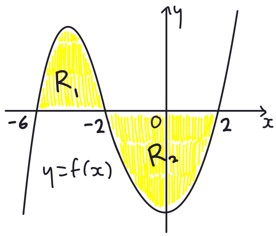
$$
\begin{array}{r|l}
 & x^2 + 8x + 12 \\
\hline
x - 2 & x^3 + 6x^2 - 4x - 24 \\
 & \underline{x^3 - 2x^2} \\
 & \quad\; 8x^2 - 4x \\
 & \quad\; \underline{8x^2 - 16x} \\
 & \qquad\quad 12x - 24 \\
 & \qquad\quad \underline{12x - 24} \\
 & \qquad\qquad\quad 0
\end{array}
$$

$$f(x) = (x-2)(x^2+8x+12)$$
$$= (x-2)(x+6)(x+2)$$

BF MATHS

c) $f(x) = (x-2)(x+6)(x+2)$

$x = 2,\ x = -6,\ x = -2$



R_1:

$$\int_{-6}^{-2} (x^3 + 6x^2 - 4x - 24)\,dx$$

$$\left[\frac{x^4}{4} + \frac{6x^3}{3} - \frac{4x^2}{2} - 24x\right]_{-6}^{-2} = \left[\frac{x^4}{4} + 2x^3 - 2x^2 - 24x\right]_{-6}^{-2}$$

Substitute $x = -2$ and $x = -6$

$$(4 - 16 - 8 + 48) - (324 - 432 - 72 + 144)$$

$R_1 = 64$

R_2:

$$\int_{-2}^{2} (x^3 + 6x^2 - 4x - 24)\,dx$$

$$\left[\frac{x^4}{4} + \frac{6x^3}{3} - \frac{4x^2}{2} - 24x\right]_{-2}^{2} = \left[\frac{x^4}{4} + 2x^3 - 2x^2 - 24x\right]_{-2}^{2}$$

Substitute $x = 2$ and $x = -2$

BF MATHS

$$(4 + 16 - 8 - 48) - (4 - 16 - 8 + 48) = -64$$

$R_2 = 64$

$$\text{Total area} = R_1 + R_2$$
$$= 64 + 64$$
$$= 128$$

BF MATHS