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Chapter 12 - Problem Solving Set A

Bronze:

a) $h(x) = 14 + 6x - x^2$

$h'(x) = 6 - 2x$

↳ $6 - 2x = 0$

$6 = 2x$

$x = 3$

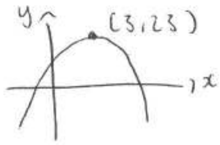
when $x = 3$

$14 + 6(3) - (3)^2$

$= 23$

$\underline{(3, 23)}$

b) $h(x) = 14 + 6x \underbrace{- x^2}_{\text{negative quadratic graph}}$



∴ this stationary point is a local maximum point

OR

$\frac{d^2y}{dx^2} = -2$

↳ $-2 < 0$

∴ local maximum point

Silver:

a) $g(x) = -x^3 - 3x^2 + 4$

$g'(x) = -3x^2 - 6x$

↳ $-3x^2 - 6x = 0$

$-3x(x+2) = 0$

$x = 0$ or $x = -2$

when $x = -2$

$-(-2)^3 - 3(-2)^2 + 4 = 0$

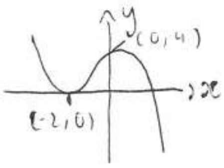
$\underline{(-2, 0)}$

when $x = 0$

$-(0)^3 - 3(0)^2 + 4 = 4$

$\underline{(0, 4)}$

b) $(-2, 0)$ touches the x axis therefore is a solution



~~$(x+2)^2(x \ldots) = -x^3 - 3x^2 + 4$~~

~~$(x^2 + 4x + 4)(x \ldots) = -x^3 - 3x^2 + 4$~~

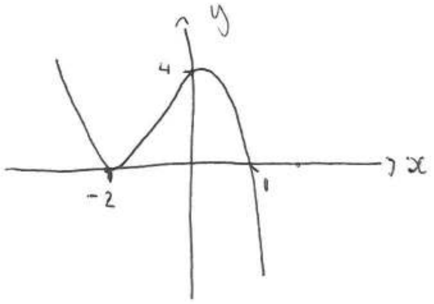
when $x = 1$

↳ $-(1)^3 - 3(1)^2 + 4 = 0$

∴ $x = 1$ is a solution

Answer: $x = -2$
$x = 1$

c)



Turning points:

$(-2, 0)$ → local minimum point

$(0, 4)$ → local maximum point

Gold:

a) $f(x) = \frac{2}{3}x^{3/2} - 10x^{1/2} - 8x^{-1/2}$

$f'(x) = x^{1/2} - 5x^{-1/2} + 4x^{-3/2}$

$\hookrightarrow x^{1/2} - 5x^{-1/2} + 4x^{-3/2} = 0$

let $y = x^{1/2}$

$y - \frac{5}{y} + \frac{4}{y^3} = 0$) $\times y^3$

$y^4 - 5y^2 + 4 = 0$

let $u = y^2$

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 4 \qquad u = 1$

$4 = y^2 \qquad 1 = y^2$

$y = \pm 2 \qquad y = \pm 1$

cannot be negative as $x^{1/2}$ can only be positive $\rightarrow$ { $x^{1/2} = 2$ $\quad$ $x^{1/2} = 1$ } $\leftarrow$ cannot be negative as $x^{1/2}$ can only be positive

$x = 4 \qquad x = 1$

when $x = 1$

$\frac{2}{3}(1)^{3/2} - 10(1)^{1/2} - 8(1)^{-1/2} = -\frac{52}{3}$ $\quad$ $\left(1, -\frac{52}{3}\right)$

when $x = 4$

$\frac{2}{3}(4)^{3/2} - 10(4)^{1/2} - 8(4)^{-1/2} = -\frac{56}{3}$ $\quad$ $\left(4, -\frac{56}{3}\right)$

b) $f'(x) = x^{1/2} - 5x^{-1/2} + 4x^{-3/2}$

$f''(x) = \frac{1}{2}x^{-1/2} + \frac{5}{2}x^{-3/2} - 6x^{-5/2}$

when $x = 1$

$\hookrightarrow \frac{1}{2}(1)^{-1/2} + \frac{5}{2}(1)^{-3/2} - 6(1)^{-5/2} = -3 \rightarrow -3 < 0$ $\therefore$ local maximum point

when $x = 4$

$\hookrightarrow \frac{1}{2}(4)^{-1/2} + \frac{5}{2}(4)^{-3/2} - 6(4)^{-5/2} = \frac{3}{8} \rightarrow \frac{3}{8} > 0$ $\therefore$ local minimum point

$\therefore \left(1, -\frac{52}{3}\right)$ is a maximum point and $\left(4, -\frac{56}{3}\right)$ is a minimum point