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12.8

1) a) $f(x) = 9x^2 - 6x + 7$

$f'(x) = 18x - 6$

$f''(x) = 18$

b) $f(x) = -x^3 + 4x^{-1}$

$f'(x) = -3x^2 - 4x^{-2}$

$f''(x) = -6x + 8x^{-3}$

c) $f(x) = 4x^{-1/2} + 8x^{3/2}$

$f'(x) = -2x^{-3/2} + 12x^{1/2}$

$f''(x) = 3x^{-5/2} + 6x^{-1/2}$

2) a) $y = 6x^{-3} - \frac{1}{2}x^4 - 9x$

$\frac{dy}{dx} = -18x^{-4} - 2x^3 - 9$

$\frac{d^2y}{dx^2} = 72x^{-5} - 6x^2$

b) $y = 8x^{-1/4} - x^{1/6} - 7$

$\frac{dy}{dx} = -2x^{-5/4} - \frac{1}{6}x^{-5/6}$

$\frac{d^2y}{dx^2} = \frac{5}{2}x^{-9/4} + \frac{5}{36}x^{-11/6}$

c) $y = 5x^2 - x^5 - 15 + 3x^3$

$y = 3x^3 + 5x^2 - x^5 - 15$

$\frac{dy}{dx} = 9x^2 + 10x - 5x^4$

$\frac{d^2y}{dx} = 18x + 10 - 20x^3$

3) a) $h(x) = \cancel{10x^5} 4x^4 - x^{1/3} + \frac{1}{x^3} - \frac{x^5}{x^3}$

$h(x) = 4x^4 - x^{1/3} + x^{-3} - x^2$

$h'(x) = 16x^3 - \frac{1}{3}x^{-2/3} - 3x^{-4} - 2x$

b) $h'(x) = 16x^3 - \frac{1}{3}x^{-2/3} - 3x^{-4} - 2x$

$h''(x) = 48x^2 + \frac{2}{9}x^{-5/3} + 12x^{-5} - 2$

4) $f(x) = x^{-1} - 6x - 2x^3$

$f'(x) = -x^{-2} - 6 - 6x^2$

$f''(x) = 2x^{-3} - 12x$

When $x = -2$

$\hookrightarrow 2(-2)^{-3} - 12(-2)$

$= 95/4$

5) a) $1^3 + \binom{3}{1}(1)^2(-4x) + \binom{3}{2}(1)(-4x)^2 + (-4x)^3$

$= 1 - 12x + 48x^2 - 64x^3$

b) $p(x) = 1 - 12x + 48x^2 - 64x^3$

$p'(x) = -12 + 96x - 192x^2$

$p''(x) = 96 - 384x$

6) $f(x) = ax^3 + bx^2 - 8x + 6$

$f'(x) = 3ax^2 + 2bx - 8$

$\hookrightarrow 3a(2)^2 + 2b(2) - 8 = -24$

$12a + 4b - 8 = -24$

$12a + 4b = -16$ (1)

$f''(x) = 6ax + 2b$

$\hookrightarrow 6a(-5) + 2b = 100$

$-30a + 2b = 100$ (11) (x2)

$-60a + 4b = 200$

$$\begin{array}{r} -60a + 4b = 200 \\ -\quad 12a + 4b = -16 \\ \hline -72a = 216 \end{array}$$

$a = 216/-72$

$a = -3$

$12(-3) + 4b = -16$

$-36 + 4b = -16$

$4b = 20$

$b = 5$

7) a) $g(x) = px^3 + qx^2 + 6x - 1$

$g'(x) = 3px^2 + 2qx + 6$

$g'(-2) = 3p(-2)^2 + 2q(-2) + 6 = 0$

$= 12p - 4q + 6 = 0$

$= 12p - 4q = -6 \quad (x-5) \rightarrow -60p + 20q = 30$

$g'(5/2) = 3p(5/2)^2 + 2q(5/2) + 6 = 0$

$= \frac{75}{4}p + 5q + 6 = 0$

$= \frac{75}{4}p + 5q = -6 \quad (x4) \rightarrow 75p + 20q = -24$

$$\begin{array}{r} 75p + 20q = -24 \\ -\ -60p + 20q = 30 \\ \hline 135p = -54 \end{array}$$

$p = -2/5$

$12(-2/5) - 4q + 6 = 0$

$-24/5 - 4q + 6 = 0$

$6/5 = 4q$

$q = 3/10$

b) $g(x) = -2/5\,x^3 + 3/10\,x^2 + 6x - 1$

$g'(x) = -6/5\,x^2 + 3/5\,x + 6$

$g''(x) = -12/5\,x + 3/5$