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13.7 Areas between curves and lines

1) a) $6 = \frac{1}{4}x^2 + 2$

$0 = \frac{1}{4}x^2 - 4$

$x = 4$ or -4

$A = (-4, 6)$

$B = (4, 6)$

b) $\int_{-4}^{4} \frac{1}{4}x^2 + 2 \, dx$

$\left[\frac{\frac{1}{4}x^3}{3} + 2x \right]_{-4}^{4} = \left[\frac{1}{12}x^3 + 2x \right]_{-4}^{4}$

$\left(\frac{1}{12}4^3 + 2(4)\right) - \left(\frac{1}{12}(-4)^3 + 2(-4)\right) = \frac{80}{3}$

c) $\int_{-4}^{4} 6 \, dx$

$\left[6x \right]_{-4}^{4}$

$\left(6(4)\right) - \left(6(-4)\right) = 48$

d) $48 - \frac{80}{3} = \frac{64}{3}$

2) a) $4 - x = -x^2 + 4x$

$0 = -x^2 + 5x - 4$

$x = 4$ or 1

$y = 4 - (4)$ $\quad$ $y = 4 - (1)$

$y = 0$ $\quad$ $y = 3$

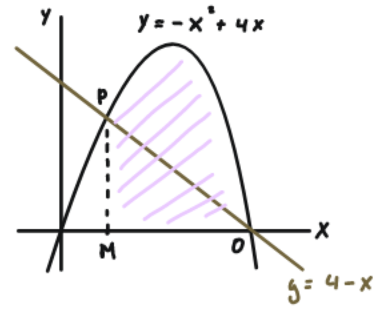
$M = (1, 3)$

$Q = (4, 0)$

b) $\int_1^4 -x^2 + 4x \; dx$

$\left[\frac{-x^3}{3} + \frac{4x^2}{2} \right]_1^4 = \left[-\frac{x^3}{3} + 2x^2 \right]_1^4$

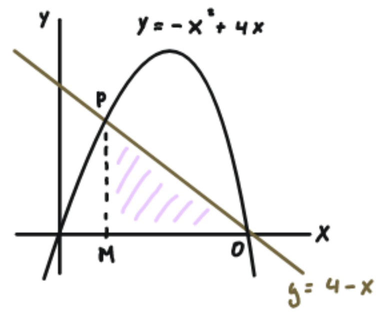
$\left(-\frac{(4)^3}{3} + 2(4)^2 \right) - \left(-\frac{(1)^3}{3} + 2(1)^2 \right) = 9$



c) $\int_1^4 4 - x \; dx$

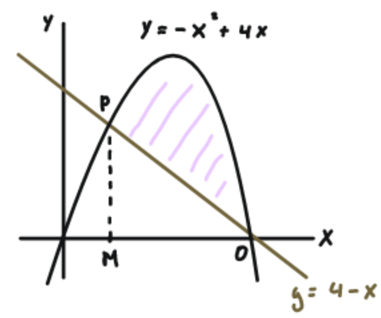
$\left[4x - \frac{x^2}{2} \right]_1^4$

$\left(4x - \frac{x^2}{2} \right) - \left(4x - \frac{x^2}{2} \right) = \frac{9}{2}$



d) $R = 9 - \frac{9}{2}$

$R = \frac{9}{2}$



3) a) $\int_9^1 4x^{1/2} - \frac{1}{2} x^{3/2} - 2 \; dx$

$\left[\frac{4x^{3/2}}{3/2} - \frac{\frac{1}{2} x^{5/2}}{5/2} - 2x \right]_1^9 = \left[\frac{8}{3} x^{3/2} - \frac{1}{5} x^{5/2} + 2x \right]_1^9$

$\left(\frac{8}{3}(9)^{3/2} - \frac{1}{5}(9)^{5/2} + 2(9) \right) - \left(\frac{8}{3}(1)^{3/2} - \frac{1}{5}(1)^{5/2} + 2(1) \right) = \frac{554}{15}$

b) $5x + 8y = 49$

$8y = -5x + 49$

$y = -\frac{5}{8} x + \frac{49}{8}$

$\int_1^9 -\frac{5}{8} x + \frac{49}{8} \; dx$

$\left[\frac{-\frac{5}{8} x^2}{2} + \frac{49}{8} x \right]_1^9 = \left[-\frac{5}{16} x^2 + \frac{49}{8} x \right]_1^9$

$\left(-\frac{5}{16}(9)^2 + \frac{49}{8}(9) \right) - \left(-\frac{5}{16}(1)^2 + \frac{49}{8}(1) \right) = 24$

$R = \frac{554}{15} - 24 \qquad R = \frac{194}{15}$

4) a) $2y = -x + 13$

$$y = -\frac{x}{2} + \frac{13}{2}$$

$$-\frac{1}{2}x^2 + \frac{9}{2}x - 4 = -\frac{x}{2} + \frac{13}{2} \quad \cdot 2$$

$$-x^2 + 9x - 8 = -x + 13$$

$$-x^2 + 10x - 21 = 0$$

$$x = 3 \quad \text{or} \quad x = 7$$

When $x = 3$ $\quad y = -\frac{3}{2} + \frac{13}{2}$

$$y = 5$$

$A = (3, 5)$

when $x = 7$ $\quad y = -\frac{7}{2} + \frac{13}{2}$

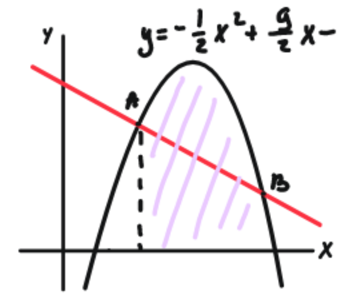
$$y = 3$$

$B = (7, 3)$

b) $$\int_3^7 -\frac{1}{2}x^2 + \frac{9}{2}x - 4 \, dx$$

$$\left[\frac{-\frac{1}{2}x^3}{3} + \frac{\frac{9}{2}x^2}{2} - 4x\right]_3^7 = \left[-\frac{1}{6}x^3 + \frac{9}{4}x^2 - 4x\right]_3^7$$

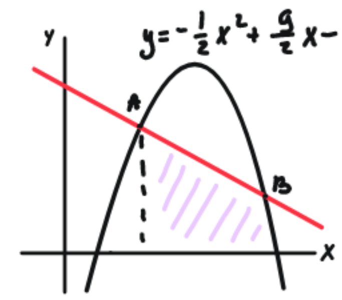
$$\left(-\frac{1}{6}(7)^3 + \frac{9}{4}(7)^2 - 4(7)\right) - \left(-\frac{1}{6}(3)^3 + \frac{9}{4}(3)^2 - 4(3)\right) = \frac{64}{3}$$



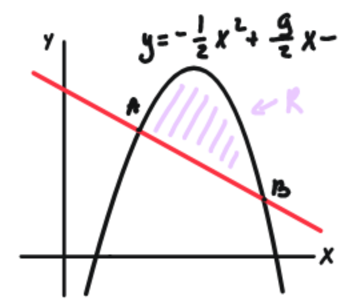
$$\int_3^7 -\frac{x}{2} + \frac{13}{2} \, dx$$

$$\left[\frac{-\frac{1}{2}x^2}{2} + \frac{13}{2}x\right]_3^7 = \left[-\frac{1}{4}x^2 + \frac{13}{2}x\right]_3^7$$

$$\left(-\frac{1}{4}(7)^2 + \frac{13}{2}(7)\right) - \left(-\frac{1}{4}(3)^2 + \frac{13}{2}(3)\right) = 16$$



$$R = \frac{64}{3} - 16 = \frac{16}{3}$$



5) $h(x) = x^2(x-4)^2$

$$x^2(x^2 - 8x + 16)$$

$$x^4 - 8x^3 + 16x^2$$

$$\int_1^2 x^4 - 8x^3 + 16x^2 \, dx$$

$$\left[\frac{x^5}{5} - \frac{8x^4}{4} + \frac{16x^3}{3} \right]_1^2 = \left[\frac{x^5}{5} - 2x^4 + \frac{16}{3}x^3 \right]_1^2$$

$$\left(\frac{2^5}{5} - 2(2)^4 + \frac{16}{3}(2)^3 \right) - \left(\frac{(1)^5}{5} - 2(1)^4 + \frac{16}{3}(1)^3 \right) = \frac{203}{15}$$

$$\int_1^2 7x + 2 \; dx$$

$$\left[\frac{7}{2}x^2 + 2x \right]_1^2$$

$$\left(\frac{7}{2}(2)^2 + 2(2) \right) - \left(\frac{7}{2}(1) - 2(1) \right) = \frac{25}{2}$$

$$PG = \frac{203}{15} - \frac{25}{2} = \frac{31}{30}$$

6) a) $5x + 4y - 21 = 0$

$$4y = -5x + 21$$

$$y = -\frac{5}{4}x + \frac{21}{4}$$

$\cdot x^2$

$$\frac{4}{x^2} = -\frac{5}{4}x + \frac{21}{4}$$

$$4 = -\frac{5}{4}x^3 + \frac{21}{4}x^2$$

$\cdot 4$

$$\frac{5}{4}x^3 - \frac{21}{4}x^2 + 4 = 0$$

$$5x^3 - 21x^2 + 16 = 0$$

b) $5x^3 - 21x^2 + 16 = 0$

$$x = -\frac{4}{5} \text{ or } 1 \text{ or } 4$$

$x = -\frac{4}{5}$ is negative $\therefore$ coordinate A

$x = 1$ Given x-coordinate of B $\therefore$ $x = 4$ is x-coordinate of C

$y = \frac{4}{x^2}$

$y = \frac{4}{4^2}$

$y = \frac{1}{4}$

$B = (4, \frac{1}{4})$

c) $5x + 4y - 21 = 0$

$4y = -5x + 21$

$y = -\frac{5}{4}x + \frac{21}{4}$

$\int_1^4 -\frac{5}{4}x + \frac{21}{4} \, dx$

$\left[\frac{-\frac{5}{4}x^2}{2} + \frac{21}{4}x\right]_1^4 = \left[-\frac{5}{8}x^2 + \frac{21}{4}x\right]_1^4$

$\left(-\frac{5}{8}(4)^2 + \frac{21}{4}(4)\right) - \left(-\frac{5}{8}(1)^2 + \frac{21}{4}(1)\right) = \frac{51}{8}$

$\int_1^4 \frac{4}{x^2} \, dx$

$4x^{-2}$

$\left[\frac{4x^{-1}}{-1}\right]_1^4 = \left[-4x^{-1}\right]_1^4$

$\left(-4(4)^{-1}\right) - \left(-4(1)^{-1}\right) = 3$

$R = \frac{51}{8} - 3 = \frac{27}{8}$

7) a) $3x + y + 6 = 0$

$y = -3x - 6$

$3y = -9x - 18$

$-9x - 18 = -x^3 - 2x^2 + 16x + 32$

$x^3 + 2x^2 - 25x - 50 = 0$

b) $x^3+2x^2-25x-50=0$

$x=-5$ or -2 or 5

Given that B has coordinates $(-2,0)$ $\therefore$ x coordinate of A is -5 $\therefore$

x coordinate of D is 5

when $x=5$ $\quad y=-3x-6$

$y=-3(5)-6$

$y=-21$

$D=(5,-21)$

c) $3y=-x^3-2x^2+16x+32$

$y=-\frac{x^3}{3}-\frac{2}{3}x^2+\frac{16}{3}x+\frac{32}{3}$

$\int_{-2}^{5}-\frac{x^3}{3}-\frac{2}{3}x^2+\frac{16}{3}x+\frac{32}{3}\,dx$

$\left[\frac{-\frac{1}{3}x^4}{4}-\frac{\frac{2}{3}x^3}{3}+\frac{\frac{16}{3}x^2}{2}+\frac{32}{3}x\right]_{-2}^{5}=\left[-\frac{1}{12}x^4-\frac{2}{9}x^3+\frac{8}{3}x^2+\frac{32}{3}x\right]_{-2}^{5}$

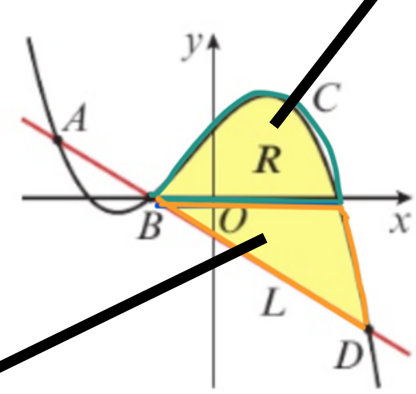
$\left(-\frac{1}{12}(5)^4-\frac{2}{9}(5)^3+\frac{8}{3}(5)^2+\frac{32}{3}(5)\right)-\left(-\frac{1}{12}(-2)^4-\frac{2}{9}(-2)^3+\frac{8}{3}(-2)^2+\frac{32}{3}(-2)\right)=\frac{1813}{36}$

$3x+y+6=0$

$y=-3x-6$

$\int -3x-6\,dx$

$\left[\frac{-3x^2}{2}-6x\right]_{-2}^{5}$



$\left(-\frac{3}{2}(5)^2-6(5)\right)-\left(-\frac{3}{2}(-2)^2-6(-2)\right)=-\frac{147}{2}$

The area is negative becauseit is under the x-axis

$R=\frac{1813}{36}-\left(-\frac{147}{2}\right)=\frac{4459}{36}$