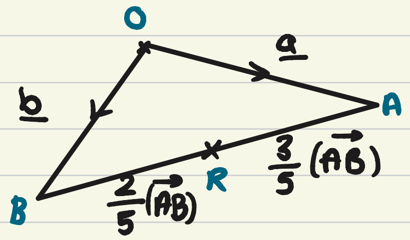
Author: Taran Vijay Limani

## Bronze

a) Show that $\overrightarrow{OP} = 2\underline{a} + 3\underline{b}$




$\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR}$

$\underline{a} + \frac{3}{5}(\overrightarrow{AB})$

to find $\overrightarrow{AR}$ you need to find $\overrightarrow{AB}$:

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -\underline{a} + b$

$\underline{a} + \frac{3}{5}\underline{b} - \frac{3}{5}\underline{a}$

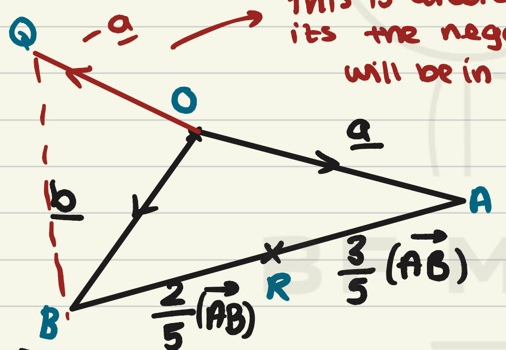
$= \frac{2}{5}\underline{a} + \frac{3}{5}\underline{b}$

$5\left(\frac{2}{5}\underline{a} + \frac{3}{5}\underline{b}\right)$

$\overrightarrow{OP} = 2\underline{a} + 3\underline{b}$

b) Prove that the points Q, B, P are collinear:

→ if co-linear they have same scalar multiple of the other




this is where point Q is because its the negative of $\overrightarrow{OA}$ so it will be in opposite direction to $\overrightarrow{OA}$

Find $\overrightarrow{QB} = \overrightarrow{QO} + \overrightarrow{OB}$

$+\underline{a} \quad +\underline{b}$

$= \underline{a} + \underline{b}$

used from earlier in question

Find $\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP}$

$\underline{a} + 2\underline{a} + 3\underline{b}$

$= 3\underline{a} + 3\underline{b}$

$= 3(\underline{a} + \underline{b})$

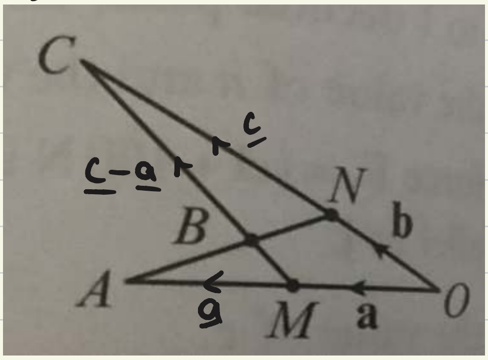
$\left.\begin{aligned}\overrightarrow{QP} &= 3(\underline{a}+\underline{b}) \\ \overrightarrow{QB} &= (\underline{a}+\underline{b})\end{aligned}\right\}$ vector QP is a scalar multiple of 3 to vector QB

$\therefore \overrightarrow{QP} = 3\overrightarrow{QB}$

$\therefore$ points Q,B,P are co-linear

Silver

a) Find value of k




$\overrightarrow{MA} = \underline{a}$

let $\overrightarrow{OC} = c$

Find $\overrightarrow{MC}$:

$\overrightarrow{MC} = \overrightarrow{MO} + \overrightarrow{OC}$

$-\underline{a} + \underline{c}$

$\overrightarrow{MC} = \underline{c} - \underline{a}$

Find $\overrightarrow{MB}$ : $\frac{1}{5}(\overrightarrow{MC})$

$\therefore \frac{1}{5}(\underline{c} - \underline{a})$

Find $\overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB}$

$\underline{a} + \frac{1}{5}(\underline{c} - \underline{a})$

$= a + \frac{1}{5}\underline{c} - \frac{1}{5}\underline{a}$

$\overrightarrow{OB} = \frac{4}{5}\underline{a} + \frac{1}{5}\underline{c}$

Find $\overrightarrow{AB}$ :

$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB}$

$-a + \frac{1}{5}\underline{c} - \frac{1}{5}\underline{a}$

$\overrightarrow{AB} = \frac{1}{5}\underline{c} - \frac{6}{5}\underline{a}$

Find $\overrightarrow{AN}$

$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$

$\overrightarrow{AN} = -2\underline{a} + \underline{b}$

Given that $\overrightarrow{AB} = k(\overrightarrow{AN})$

$\frac{1}{5}\underline{c} - \frac{6}{5}\underline{a} = k(-2\underline{a} + \underline{b})$

$-\frac{6}{5}\underline{a} = -2k\underline{a}$

$\frac{6}{5}\cancel{\underline{a}} = 2k\cancel{\underline{a}}$

$\frac{6}{5} = 2k$

$k = \frac{3}{5}$

b) Write $\overrightarrow{OC}$ as scalar multiple of $\underline{b}$

$\overrightarrow{ON} = \underline{b}$

Since N lies on $\overrightarrow{OC}$

$\overrightarrow{ON} = k\,\overrightarrow{OC}$

$\underline{b} = k\,\overrightarrow{OC}$

from part a, $k = \frac{3}{5}$

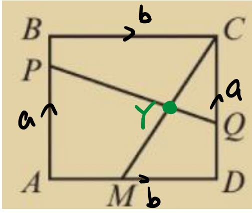
$b = \frac{3}{5}\,\overrightarrow{OC}$

make $\overrightarrow{OC}$ subject $\therefore \div$ by $\frac{3}{5}$ on both sides

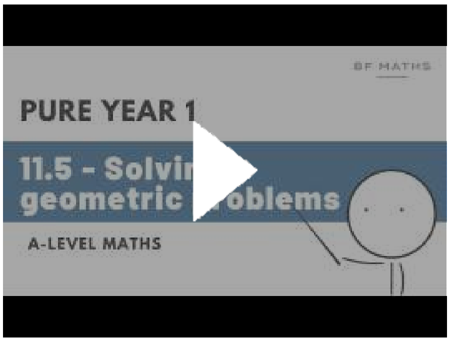
$\overrightarrow{OC} = \frac{5}{3}\,\underline{b}$

$$\overrightarrow{OC} = \frac{5}{3}\,\underline{b}$$

# Chapter 11 - Problem Solving Set B




Explanation on similar proof question:
[A-Level Maths | Pure Year 1 | 11.5 - Solving geometric problems with vectors Walkthrough | Edexcel]




**Gold**

Let $\overrightarrow{AB} = \underset{\sim}{a}$

$\overrightarrow{BC} = \underset{\sim}{b}$

$$\begin{cases} \overrightarrow{PY} = \lambda(\overrightarrow{PQ}) \\ \overrightarrow{MY} = \mu(\overrightarrow{MC}) \end{cases}$$

If $\overrightarrow{PQ}$ bisects $\overrightarrow{MC}$ then $\mu$ should equal to $\frac{1}{2}$

$$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BC} + \overrightarrow{CQ}$$
$$= \frac{1}{5}a + b + \left(-\frac{3}{5}a\right)$$
$$= -\frac{2}{5}a + b$$

$$\overrightarrow{MC} = \overrightarrow{MD} + \overrightarrow{DC}$$
$$= \frac{1}{2}b + a$$

$$\Rightarrow \begin{cases} \overrightarrow{PY} = \lambda\left(-\frac{2}{5}a + b\right) = -\frac{2}{5}\lambda a + \lambda b \\ \overrightarrow{MY} = \mu\left(\frac{1}{2}b + a\right) = \frac{1}{2}\mu b + \mu a \end{cases}$$

$$\because \overrightarrow{PY} = \overrightarrow{PA} + \overrightarrow{AM} + \overrightarrow{MY}$$

$$\therefore -\frac{2}{5}\lambda a + \lambda b = -\frac{4}{5}a + \frac{1}{2}b + \frac{1}{2}\mu b + \mu a$$

$$-\frac{2}{5}\lambda a + \lambda b = \left(-\frac{4}{5} + \mu\right)a + \left(\frac{1}{2} + \frac{1}{2}\mu\right)b$$

$$\Rightarrow \begin{cases} -\frac{2}{5}\lambda = -\frac{4}{5} + \mu \\ \lambda = \frac{1}{2} + \frac{1}{2}\mu \end{cases} \xrightarrow[\text{equations}]{\text{simultaneous}} -\frac{2}{5}\left(\frac{1}{2} + \frac{1}{2}\mu\right) = -\frac{4}{5} + \mu$$

$$-\frac{1}{5} - \frac{1}{5}\mu = -\frac{4}{5} + \mu$$

$$\frac{3}{5} = \frac{6}{5}\mu$$

$$3 = 6\mu$$

$$\mu = \frac{1}{2}$$

$$\therefore \overrightarrow{MY} = \frac{1}{2}\overrightarrow{MC}$$

$$\therefore \overrightarrow{PQ} \text{ bisects } \overrightarrow{MC}$$