

Author: Taran Vijay Limani

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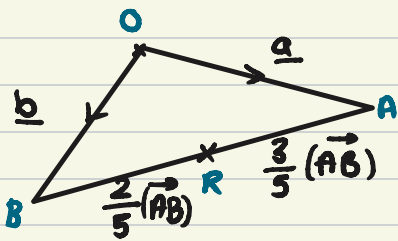
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BF MATHS

Thank you for using BF Maths for your maths revision!

Bronze

a) Show that $\vec{OP} = 2\vec{a} + 3\vec{b}$



$$\vec{OR} = \vec{OA} + \vec{AR}$$

$$\vec{a} + \frac{2}{5}(\vec{AB})$$

to find \vec{AR} you need to find \vec{AB} :

$$\vec{AB} = \vec{AO} + \vec{OB}$$

$$\vec{AB} = -\vec{a} + \vec{b}$$

$$\vec{a} + \frac{2}{5}\vec{b} - \frac{2}{5}\vec{a}$$

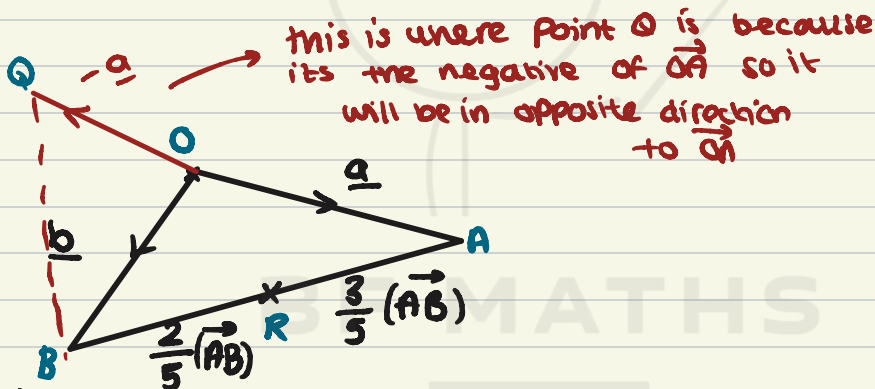
$$= \frac{2}{5}\vec{a} + \frac{2}{5}\vec{b}$$

$$5\left(\frac{2}{5}\vec{a} + \frac{2}{5}\vec{b}\right)$$

$$\vec{OP} = 2\vec{a} + 3\vec{b}$$

b) Prove that the points Q, B, P are collinear:

→ if co-linear they have same scalar multiple of the other



$$\text{Find } \vec{QB} = \vec{QO} + \vec{OB}$$

$$= -\vec{a} + \vec{b}$$

$$= \vec{b} - \vec{a}$$

$$\text{Find } \vec{QP} = \vec{QO} + \vec{OP}$$

$$= -\vec{a} + 2\vec{a} + 3\vec{b}$$

$$= \vec{a} + 3\vec{b}$$

$$= 3(\vec{b} - \vec{a})$$

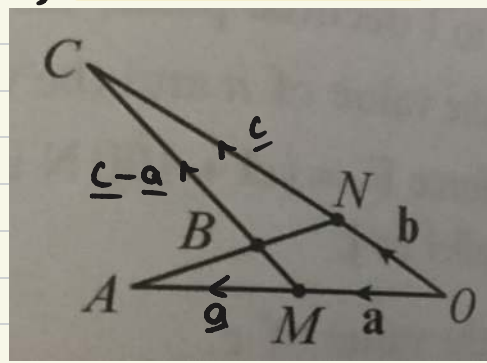
$$\left. \begin{aligned} \vec{QP} &= 3(\vec{b} - \vec{a}) \\ \vec{QB} &= (\vec{b} - \vec{a}) \end{aligned} \right\} \begin{array}{l} \text{vector } \vec{QP} \text{ is a} \\ \text{scalar multiple of 3 to vector } \vec{QB} \end{array}$$

$$\therefore \vec{QP} = 3\vec{QB}$$

∴ points Q, B, P are co-linear

Silver

a) Find value of k



$$\overrightarrow{MA} = \underline{a}$$
$$\text{let } \overrightarrow{OC} = \underline{c}$$

Find \overrightarrow{MC} :

$$\overrightarrow{MC} = \overrightarrow{MO} + \overrightarrow{OC}$$

$$\overrightarrow{MC} = \underline{-a} + \underline{c}$$

$$\text{Find } \overrightarrow{MB} : \frac{1}{5} (\overrightarrow{MC})$$

$$\therefore \frac{1}{5} (\underline{c} - \underline{a})$$

$$\text{Find } \overrightarrow{OB}$$
$$\overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB}$$

$$\underline{a} + \frac{1}{5} (\underline{c} - \underline{a})$$
$$= \underline{a} + \frac{1}{5} \underline{c} - \frac{1}{5} \underline{a}$$

$$\overrightarrow{OB} = \frac{4}{5} \underline{a} + \frac{1}{5} \underline{c}$$

$$\text{Find } \overrightarrow{AB} :$$
$$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB}$$

$$-\underline{a} + \frac{1}{5} \underline{c} - \frac{1}{5} \underline{a}$$

$$\overrightarrow{AB} = \frac{1}{5} \underline{c} - \frac{6}{5} \underline{a}$$

Find \overrightarrow{AN}

$$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$$

$$\overrightarrow{AN} = -2\underline{a} + \underline{b}$$

Given that $\overrightarrow{AB} = k(\overrightarrow{AN})$

$$\frac{1}{5} \underline{c} - \frac{6}{5} \underline{a} = k(-2\underline{a} + \underline{b})$$

$$-\frac{6}{5} \underline{a} = -2k\underline{a}$$

$$\frac{6}{5} \underline{a} = 2k\underline{a}$$

$$\frac{6}{5} = 2k$$

$$k = \frac{3}{5}$$

b) Write \vec{OC} as scalar multiple of \underline{b}

$$\vec{ON} = \underline{b}$$

Since N lies on \vec{OC}

$$\vec{ON} = k \vec{OC}$$

$$\underline{b} = k \vec{OC}$$

from part a, $k = \frac{3}{5}$

$$\underline{b} = \frac{3}{5} \vec{OC}$$

make \vec{OC} subject $\therefore \div$ by $\frac{3}{5}$ on both sides

$$\vec{OC} = \frac{5}{3} \underline{b}$$

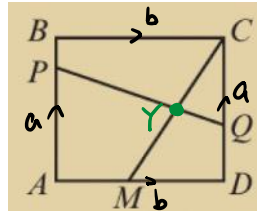
$$\vec{OC} = \frac{5}{3} \underline{b}$$



BF MATHS

Chapter 11 - Problem Solving Set B

Gold



$$\text{Let } \vec{AB} = \vec{a}$$

$$\vec{BC} = \vec{b}$$

$$\begin{cases} \vec{PY} = \lambda(\vec{PQ}) \\ \vec{MY} = \mu(\vec{MC}) \end{cases}$$

If \vec{PQ} bisects \vec{MC} then μ should equal to $\frac{1}{2}$

$$\begin{aligned} \vec{PQ} &= \vec{PB} + \vec{BC} + \vec{CQ} \\ &= \frac{1}{5}\vec{a} + \vec{b} + (-\frac{3}{5}\vec{a}) \\ &= -\frac{2}{5}\vec{a} + \vec{b} \end{aligned}$$

$$\begin{aligned} \vec{MC} &= \vec{MD} + \vec{DC} \\ &= \frac{1}{2}\vec{b} + \vec{a} \end{aligned}$$

$$\Rightarrow \begin{cases} \vec{PY} = \lambda(-\frac{2}{5}\vec{a} + \vec{b}) = -\frac{2}{5}\lambda\vec{a} + \lambda\vec{b} \\ \vec{MY} = \mu(\frac{1}{2}\vec{b} + \vec{a}) = \frac{1}{2}\mu\vec{b} + \mu\vec{a} \end{cases}$$

$$\therefore \vec{PY} = \vec{PA} + \vec{AM} + \vec{MY}$$

$$\therefore -\frac{2}{5}\lambda\vec{a} + \lambda\vec{b} = -\frac{4}{5}\vec{a} + \frac{1}{2}\vec{b} + \frac{1}{2}\mu\vec{b} + \mu\vec{a}$$

$$-\frac{2}{5}\lambda\vec{a} + \lambda\vec{b} = (-\frac{4}{5} + \mu)\vec{a} + (\frac{1}{2} + \frac{1}{2}\mu)\vec{b}$$

$$\Rightarrow \begin{cases} -\frac{2}{5}\lambda = -\frac{4}{5} + \mu \\ \lambda = \frac{1}{2} + \frac{1}{2}\mu \end{cases}$$

Simultaneous equations \rightarrow

$$-\frac{2}{5}(\frac{1}{2} + \frac{1}{2}\mu) = -\frac{4}{5} + \mu$$

$$-\frac{1}{5} - \frac{1}{5}\mu = -\frac{4}{5} + \mu$$

$$\frac{3}{5} = \frac{6}{5}\mu$$

$$3 = 6\mu$$

$$\mu = \frac{1}{2}$$

$$\therefore \vec{MY} = \frac{1}{2}\vec{MC}$$

$$\therefore \vec{PQ} \text{ bisects } \vec{MC}$$

Explanation on similar proof question:

[A-Level Maths | Pure Year 1 | 11.5 - Solving geometric problems with vectors Walkthrough | Edexcel](#)

