Author: <u>Brunelle Ndongala</u>

This step-by-step solution guide has been created by **Brunelle Ndongala** for educational purposes. While we have made every effort to ensure the accuracy of the information presented, it is possible that there may be errors or omissions. We encourage users to critically evaluate and verify the content. BF Maths and the author cannot be held responsible for any errors or inaccuracies in this guide.

If you find any mistakes or have any suggestions for improvements, please contact us at bfmathshello@gmail.com. Your feedback is invaluable in helping us maintain the quality and accuracy of our resources. Please specify *which exercise and which question* in the email.

Thank you for using BF Maths for your maths revision!

12.9 Stationary points

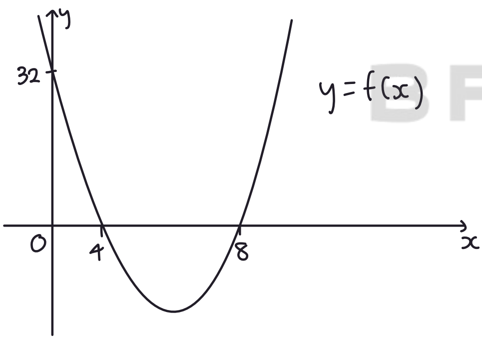
1.

a) $f(x) = (x-4)(x-8)$

when $y = 0$
$x = 4$ or $x = 8$

when $x = 0$
$f(x) = 32$

Sketch graph:



b) The curve $y=f(x)$ changes from being a decreasing function to an increasing function between $x=4$ and $x=8$, so the curve has a local minimum in this interval

c) $f(x) = (x-4)(x-8)$
$= x^2 - 12x + 32$

$f'(x) = 2x - 12$

d) $f'(x) = 0$
$2x - 12 = 0$
$2x = 12$
$x = 6$

e) Substitute $x=6$ into $f(x)$
↳ $6^2-12(6)+32=-4$

Coordinate of stationary point $=(6,-4)$

2.

a) $y=x^2-6x+8$

Differentiate: $\frac{dy}{dx}=2x-6$

When $\frac{dy}{dx}=0$: $2x-6=0$
$2x=6$
$x=3$

Substitute $x=3$ into y: $3^2-6(3)+8=-1$

Coordinate of minimum point $=(3,-1)$

b) $y=x^2-3x-11$

Differentiate: $\frac{dy}{dx}=2x-3$

When $\frac{dy}{dx}=0$: $2x-3=0$
$2x=3$
$x=\frac{3}{2}$

Substitute $x=\frac{3}{2}$ into y: $\left(\frac{3}{2}\right)^2-3\left(\frac{3}{2}\right)-11=-\frac{53}{4}$

Coordinate of minimum point $=\left(\frac{3}{2},-\frac{53}{4}\right)$

c) $y = 3x^2 + 2x + 1$

Differentiate: $\frac{dy}{dx} = 6x + 2$

When $\frac{dy}{dx} = 0$: $6x + 2 = 0$
$6x = -2$
$x = -\frac{1}{3}$

Substitute $x = -\frac{1}{3}$ into y: $3\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right) + 1 = \frac{2}{3}$

Coordinate of minimum point $= \left(-\frac{1}{3}, \frac{2}{3}\right)$

3.

a) $y = 8 - 16x - x^2$

Differentiate: $\frac{dy}{dx} = -16 - 2x$

When $\frac{dy}{dx} = 0$: $-16 - 2x = 0$
$-2x = 16$
$-x = 8$
$x = -8$

Substitute $x = -8$ into $y = 8 - 16(-8) - (-8)^2 = 72$

Coordinate of maximum point $= (-8, 72)$

b) $y = 12x - 4x^2$

Differentiate: $\frac{dy}{dx} = 12 - 8x$

When $\frac{dy}{dx} = 0$: $12 - 8x = 0$

$$-8x = -12$$
$$8x = 12$$
$$x = \frac{3}{2}$$

Substitute $x = \frac{3}{2}$ into y: $12\left(\frac{3}{2}\right) - 4\left(\frac{3}{2}\right)^2 = 9$

Coordinate of maximum point $= \left(\frac{3}{2}, 9\right)$

c) $y = 18 - 30x - 7x^2$

Differentiate: $\frac{dy}{dx} = -30 - 14x$

When $\frac{dy}{dx} = 0$: $-30 - 14x = 0$

$$-14x = 30$$
$$14x = -30$$
$$x = -\frac{15}{7}$$

Substitute $x = -\frac{15}{7}$ into y: $18 - 30\left(-\frac{15}{7}\right) - 7\left(-\frac{15}{7}\right)^2 = \frac{351}{7}$

Coordinate of maximum point $= \left(-\frac{15}{7}, \frac{351}{7}\right)$

4.

a) $y = -4x^3 + 6x^2 + 24x + 3$

$\frac{dy}{dx} = -12x^2 + 12x + 24$

When $\frac{dy}{dx} = 0$: $-12x^2 + 12x + 24 = 0$

$x = 2$ or -1

Sub $x=2$ into y:

$-4(2)^3 + 6(2)^2 + 24(2) + 3$

$= 43$

Sub $x=-1$ into y:

$-4(-1)^3 + 6(-1)^2 + 24(-1) + 3$

$= -11$

Stationary points $=$ $(2, 43)$ and $(-1, -11)$

b) $\frac{dy}{dx} = -12x^2 + 12x + 24$

$\frac{d^2y}{dx^2} = -24x + 12$

c) Stationary points $=$ $(2, 43)$ and $(-1, -11)$

When $x = 2$

$\frac{d^2y}{dx^2} = -24(2) + 12 = -36 < 0$

so $(2, 43)$ is a local maximum

When $x = -1$

$\frac{d^2y}{dx^2} = -24(-1) + 12 = 36 > 0$

so $(-1, -11)$ is a local minimum

5. $q(x) = -x^3 - 6x^2 - 12x - 6$

a) $q'(x) = -3x^2 - 12x - 12$

When $q'(x) = 0$: $-3x^2 - 12x - 12 = 0$
$x = -2$

Sub $x = -2$ into $y = q(x)$: $-(-2)^3 - 6(-2)^2 - 12(-2) - 6 = 2$

Stationary point $= (-2, 2)$

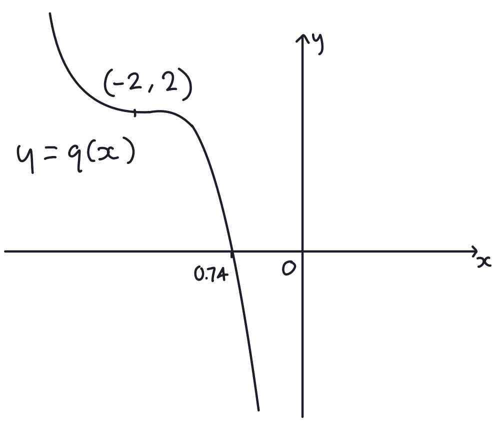
b) $q''(x) = -6x - 12$

When $x = -2$: $q''(x) = -6(-2) - 12$
$= 0$

gradient of curve at $x = -2.1$ and $x = -1.9$ are both negative so $(-2, 2)$ is a point of inflection

c) $q(x) = -x^3 - 6x^2 - 12x - 6$

negative graph
point of inflection at $(-2, 2)$
crosses x-axis at $x = -0.74$



6. $f(x) = 2x^3 - \frac{1}{2}x^2 - x + 2$

$f'(x) = 6x^2 - x - 1$

when $f'(x) = 0$: $6x^2 - x - 1 = 0$

$x = \frac{1}{2}$ or $x = -\frac{1}{3}$

Sub $x = \frac{1}{2}$ into $y = f(x)$

$2\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 2 = \frac{13}{8}$

Sub $x = -\frac{1}{3}$ into $y = f(x)$

$2\left(-\frac{1}{3}\right)^3 - \frac{1}{2}\left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 2 = \frac{119}{54}$

Stationary points $= \left(\frac{1}{2}, \frac{13}{8}\right)$ and $\left(-\frac{1}{3}, \frac{119}{54}\right)$

7. $g(x) = \frac{54}{x} + 4\sqrt{x}$

a) $g(x) = 54x^{-1} + 4x^{\frac{1}{2}}$

$g'(x) = -54x^{-2} + 2x^{-\frac{1}{2}}$

when $g'(x) = 0$: $-54x^{-2} + 2x^{-\frac{1}{2}} = 0$

$$\frac{-54}{x^2} + \frac{2}{\sqrt{x}} = 0$$

$$\frac{2}{\sqrt{x}} = \frac{54}{x^2}$$

$2x^2 = 54\sqrt{x}$

let $u = \sqrt{x}$, then $u^2 = x$

$2u^4 = 54u$

$2u^3 = 54$

$u^3 = 27$

$u = 3$

thus, $\sqrt{x} = 3$ and $x = 9$

Substitute $x=9$ into $y=g(x)$:

$$g(9) = \frac{54}{9} + 4\sqrt{9}$$
$$= 18$$

Coordinates of stationary point $= (9, 18)$

b) $g'(x) = -54x^{-2} + 2x^{-\frac{1}{2}}$

$g''(x) = 108x^{-3} - x^{-\frac{3}{2}}$

c) Sub $x=9$ into $g''(x)$:

$$g''(x) = 108(9)^{-3} - (9)^{-\frac{3}{2}}$$
$$= \frac{1}{9}$$

$\frac{1}{9} > 0$ so $(9, 18)$ is a local minimum

8. $f(x) = -2x^3 + 4x^2 + 8x - 10$

$f'(x) = -6x^2 + 8x + 8$

when $f'(x) = 0$: $-6x^2 + 8x + 8 = 0$

$x = 2$ or $x = -\frac{2}{3}$

Sub $x=2$ into $y=f(x)$

$-2(2)^3 + 4(2)^2 + 8(2) - 10$
$= 6$

Sub $x=-\frac{2}{3}$ into $y=f(x)$

$-2\left(-\frac{2}{3}\right)^3 + 4\left(-\frac{2}{3}\right)^2 + 8\left(-\frac{2}{3}\right) - 10$

$= -\frac{350}{27}$

Stationary points $= (2,6)$ and $\left(-\frac{2}{3}, -\frac{350}{27}\right)$

$f''(x) = -12x + 8$

When $x = -\frac{2}{3}$: $-12\left(-\frac{2}{3}\right) + 8 = 16 > 0$

when $x = 2$: $f''(x) = -12(2) + 8 = -16 < 0$

so $(2,6)$ is a local maximum

9. $h(x) = x^3 - 3x^2 + 3x + 4$

a) $h'(x) = 3x^2 - 6x + 3$

when $h'(x) = 0$: $3x^2 - 6x + 3 = 0$
$x = 1$

Sub $x=1$ into $y = h(x)$: $1^3 - 3(1)^2 + 3(1) + 4 = 5$

Stationary point $= (1,5)$

$h''(x) = 6x - 6$

when $x = 1$: $h''(x) = 6(1) - 6$
$= 0$

gradient of curve at $x = 0.9$ and $x = 1.1$ are both positive
so $(1,5)$ is a point of inflection

b) $h(x) = x^3 - 3x^2 + 3x + 4$

positive graph
point of inflection at $(1, 5)$
crosses x-axis at -0.71