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$$\textcircled{1} \text{ a) } 6i - 8j$$

$$|v| = \sqrt{6^2 + (-8)^2} \\ = 10.$$

$$\text{b) } 5i + 4j$$

$$|v| = \sqrt{5^2 + 4^2} \\ = \sqrt{41}$$

$$\text{c) } -3i + 6j$$

$$|v| = \sqrt{(-3)^2 + 6^2} \\ = 3\sqrt{5}$$

$$\text{d) } -8i - 8j$$

$$|v| = \sqrt{(-8)^2 + (-8)^2} \\ = 8\sqrt{2}$$

$$\textcircled{2} \text{ a) } 2i + 4j$$

$$\theta = \tan^{-1}\left(\frac{4}{2}\right) \\ \theta = \tan^{-1}(2) \\ \theta \approx 63.4^\circ$$

$$\text{b) } -7i - j$$

$$\theta = \tan^{-1}\left(\frac{-1}{-7}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{7}\right)$$

$$\theta \approx 8.1$$

$$\text{c) } 6j = 0i + 6j$$

$$\theta = 90^\circ$$

(the vector is in Quadrant 3, so we ^{subtract} add 180)

$$\theta = 180^\circ - 8.1^\circ$$

$$= 171.9^\circ$$

$$\text{d) } 5i - 5j$$

$$\theta = \tan^{-1}\left(\frac{-5}{5}\right)$$

$$= \tan^{-1}(-1)$$

$$\theta = 45^\circ$$

As the vector is in Quadrant 4, the answer has to be $\theta = 45^\circ$ below axis.

③ find the unit vector in the direction $-5i + 12j$

$$u = \frac{v}{|v|}$$

$$u = \left(\frac{-5}{13}\right)i + \left(\frac{12}{13}\right)j$$

$$v = -5i + 12j$$

$$|v| = \sqrt{(-5)^2 + 12^2}$$

$$= 13$$

$$u = -\frac{5}{13}i + \frac{12}{13}j$$

④ $p = 50 \cos 45^\circ = 50 \times \frac{\sqrt{2}}{2}$

$$= 25\sqrt{2}$$

the vector -
positive x direction
& negative y direction

$$q = 50 \sin 45^\circ = 50 \times \frac{\sqrt{2}}{2}$$

$$= 25\sqrt{2}$$

$$r = 25\sqrt{2}i - 25\sqrt{2}j$$

⑤ $|a| = 11 \rightarrow 11^2 = \sqrt{p^2 + q^2}$

$$|a| = \sqrt{p^2 + q^2}$$

$$p = 11 \cos 20$$

$$= 11 \times 0.9397$$

$$= 10.3$$

$$q = 11 \sin 20$$

$$= 11 \times 0.3420$$

$$= 3.76$$

$$p = 10.3$$

$$q = 3.76$$

⑥ $2|p| + 3|q| = \sqrt{13}$

$$\sqrt{p^2 + q^2} = 7$$

$$p^2 + q^2 = 49$$

$$q^2 = 49 - p^2$$

$$2|p| + 3\sqrt{49 - p^2} = \sqrt{13}$$

$$3\sqrt{49 - p^2} = \sqrt{13} - 2|p|$$

$$|q| = \sqrt{49 - p^2} \rightarrow 9(49 - p^2) = (\sqrt{13} - 2|p|)^2$$

$$441 - 9p^2 = 13 - 4\sqrt{13}|p| + 4p^2$$

Q6 continued:

$$441 - 13 = 9p^2 + 4p^2 - 4\sqrt{13}|p|$$

$$428 = 13p^2 - 4\sqrt{13}|p|$$

$$428 = 13p^2 - 4(3.605)|p|$$

$$p = \pm 7$$

(7) a) $\vec{OA} = 5\vec{i} + 4\vec{j}$, $\vec{AB} = 7\vec{i} - 4\vec{j}$, $\vec{CB} = 10\vec{i} + 7\vec{j}$

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$= (5\vec{i} + 4\vec{j}) + (7\vec{i} - 4\vec{j})$$
$$= 12\vec{i} + 0\vec{j}$$

$$\cos \theta = \frac{a \cdot b}{|a||b|}$$

c) $\angle AOB = 38.7^\circ$

$$\vec{OA} \times \vec{OB} = (5 \times 12) + (4 \times 0)$$
$$= 60$$

$$|\vec{OA}| = \sqrt{5^2 + 4^2}$$
$$= \sqrt{41}$$

$$|\vec{OB}| = \sqrt{12^2 + 0^2}$$
$$\Rightarrow \sqrt{144} = 12$$

$$\cos \theta = \frac{60}{\sqrt{41} \times 12}$$

$$\theta = 38.7^\circ$$

b) $\vec{OC} = \vec{OB} + \vec{CB}$

$$= 12\vec{i} + 10\vec{i} + 7\vec{j}$$
$$= 22\vec{i} + 7\vec{j}$$

d) $\angle BOC$

$$\vec{OB} \times \vec{OC} = (12 \times 7) + (0 \times 22)$$
$$= 84$$

$$|\vec{OB}| = 12$$
$$|\vec{OC}| = \sqrt{22^2 + 7^2}$$
$$= \sqrt{533}$$

$$\cos \theta = \frac{84}{12 \times \sqrt{533}}$$

$$\theta = 74.1^\circ$$

$$\angle AOC = \angle AOB + \angle BOC$$
$$= 38.7^\circ + 74.1^\circ$$
$$= 112.7^\circ$$

Final answers =

$$\angle AOB = 38.7^\circ$$

$$\angle BOC = 74.1^\circ$$

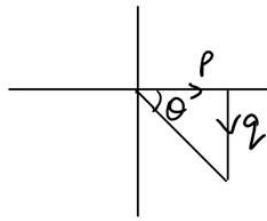
$$\angle AOC = 112.7^\circ$$

$$8) \sqrt{p^2 + q^2} = 6.5$$

$$p^2 + q^2 = 42.25$$

$$\tan \theta = -\frac{5}{12} = \frac{q}{p}$$

← opp
← adj



$$\Rightarrow q = -\frac{5}{12}p$$

Substitute into $p^2 + q^2 = 42.25$

$$p^2 + \left(-\frac{5}{12}p\right)^2 = 42.25$$

$$p^2 + \frac{25}{144}p^2 = 42.25$$

$$p^2 \left(1 + \frac{25}{144}\right) = 42.25$$

$$p^2 \left(\frac{169}{144}\right) = 42.25$$

$$p^2 = 42.25 \times \frac{144}{169}$$

$$p^2 = 36$$

$$p = \pm \sqrt{36}$$

$$p = \pm 6$$

Using $q = -\frac{5}{12}p$

$$q = -\frac{5}{12} \times (\pm 6)$$

$$q = \pm 2.5$$

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