Author: Alina Prynkevych

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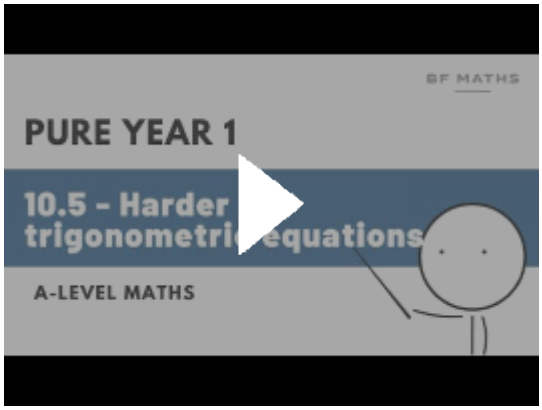
If you need help on this chapter:
A-Level Maths | Pure Year 1 | 10.5 - Harder trigonometric equations (CAST Diagram) | Edexcel

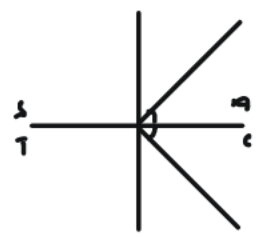


10.5 Harder trigonometric equations

1) a) let $x = 3\theta$ then $0 \leq x \leq 1080°$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30$



$x = 30, 330$

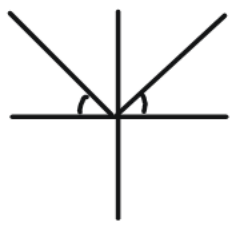
$x = 30, 330, 390, 690, 750, 1050$ ↓ ÷3

$\theta = 10, 110, 130, 230, 250, 350$

b) Let $x = \frac{\theta}{2}$ $0 \leq x \leq 180$

$\sin x = \frac{1}{\sqrt{2}}$

$x = 45$



$x = 45, 135$ ↓ x2

$\theta = 90, 270$

c) let $x = 2\theta$ $0 \leq \theta \leq 720$

$\tan x = -\sqrt{3}$

$x = -60$

$x = 120, 300, 480, 660$ ↓ x2

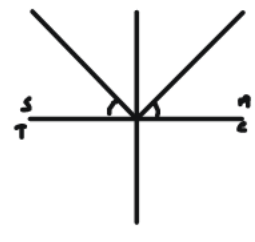
$\theta = 60, 150, 240, 330$

BF MATHS

2) a) Let $x = \theta + 30$ $30 \leq \theta + 30 < 390$

$\sin x = 0.2$

$x = 11.5$



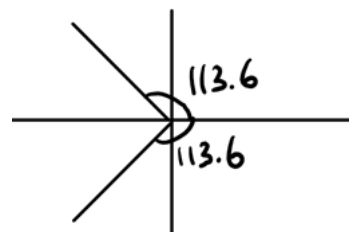
$x = 168.5, 371.5$ ↓ -30

$\theta = 138.5, 341.5$

b) Let $x = 60 - \theta$ $\quad -120 \leq 60 - \theta \leq 240$

$\cos x = -0.4$

$x = 113.6$



$60 - \theta = 113.6° \text{ or } \cancel{246.4°}$

$\downarrow -360°$

$\cancel{-246.4} \text{ or } -113.6°$

$-\theta = 53.6° \text{ or } -173.6°$

$\theta = -53.6° \text{ or } 173.6°$

c) Let $x = \theta - 20$ $\quad -20 \leq \theta - 20 \leq 340°$

$\tan x = 9$

$x = 83.7$

$x = 83.7, 263.7$

$\theta = 103.7, 283.7$ $\quad \downarrow +20$

3) a) Let $3\theta = x$ $\quad -180(3) \leq 3\theta \leq 180(3)$

$\cos = \sqrt{3} \sin$ $\quad -540 \leq 3\theta \leq 540$

$\frac{\cos x}{\cos x} = \frac{\sqrt{3} \sin x}{\cos x}$

$1 = \sqrt{3} \tan$

$\tan = \frac{1}{\sqrt{3}}$

$x = 30$

$x = -510, -330, -150, 30, 210, 390$ $\quad \downarrow \div 3$

$\theta = -170, -110, -50, 10, 70, 130$

BF MATHS

b) Let $4\theta = x$ $\quad 0 \leq 4\theta \leq 1440$

$2\sin x - 5\cos x = 0$ $\quad x = 68.2, 248.2, 428.2, 608.2,$

$788.2, 968.2, 1148.2, 1328.2$

$\frac{2\sin x}{\cos x} = \frac{5\cos x}{\cos x}$

$\theta = 17.0, 62.0, 107.0, 152.0, 197.0$

$242.0, 287.0, 332.0$

$2\tan x = 5$

$\tan x = \frac{5}{2}$

$x = 68.2$

c) let $x = \theta + 40$ $\quad -140 \leq \theta + 40 \leq 220$

$5\sin x + 8\cos x = 0$

$\frac{5\sin x}{\cos x} = \frac{-8\cos x}{\cos x}$

$5\tan x = -8$

$\tan x = -\frac{8}{5}$

$x = -58.0, 122$ $\quad -40$

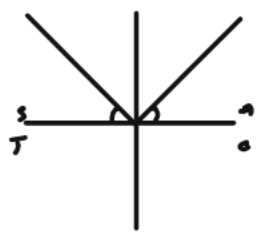
$\theta = -98, 82$

4) a) Let $x = 2x - 30$ $\quad 0 \leq 2x - 30 \leq 2 \times 360 - 30$

$0 \leq 2x - 30 \leq 690$

$\sin x = 0.75$

$x = 48.6$



$x = 48.6, 131, 4$

$x = 39.3, 80.7, 219.3, 260.7$

$2x - 30 = 48.6, 131.4, 408.6, 491.4$

Example: $2x - 30 = 48.6$

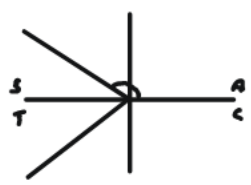
$2x = 78.6$

$x = 39.3$

b) $0 \leq 3x + 20 \leq 3(180) + 20$

$0 \leq 3x + 20 \leq 560$

$\sin^{-1}(-0.4) = 113.6$



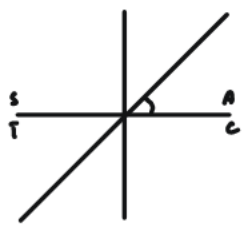
$3x + 20 = 113.6, 246.4, 473.6$

$x = 31.2, 75.5, 151.2$

BF MATHS

c) $0 \leq 4x - 45 \leq 4(180) - 45$

$0 \leq 4x - 45 \leq 675$



$\tan^{-1}(2) = 63.4$

$4x - 45 = 63.4, 243.4, 423.4, 603.4$

$x = 27.1, 72.1, 117.1, 162.1$

d) $\frac{2\sin(2x+10)}{\cos(2x+10)} = \frac{3\cos(2x+10^\circ)}{\cos(2x+10)}$

$2\tan(2x+10) = 3$

$\tan(2x+10) = \frac{3}{2}$

$2x + 10 = 56.3$

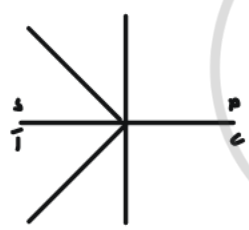
$2(-180) + 10 \leq 2x + 10 \leq 2(180) + 10$

$-350 \leq 2x + 10 \leq 370$

$2x + 10 = -303.7, -123.7, 56.3, 236.3$

$x = -156.8, -66.8, 23.2, 113.2$

5) a) $0 \leq 3x \leq 1080$



$\cos x = -\frac{1}{2}$

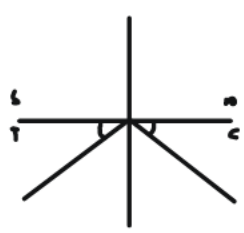
$3x = 120, 240, 480, 600, 840, 960$

$x = 40^\circ, 80^\circ, 160^\circ, 200^\circ, 280^\circ, 320^\circ$

b) $0 \leq x - 20 \leq 340$

$\sin(x - 20) = -\frac{1}{\sqrt{2}}$

$x = -45$



$x - 20 = 225, 315$

$x = 245^\circ, 335^\circ$

6) $0 \leq 2x \leq 720$

$\frac{5\sin(2x)}{\cos} = \frac{3\cos(2x)}{\cos}$

$5\tan(2x) = 3$

$\tan(2x) = \frac{3}{5}$

$2x = 30.96, 210.96, 390.96, 570.96, 750.96$

$x = 15.48, 105.48, 195.48, 285.5$

BF MATHS

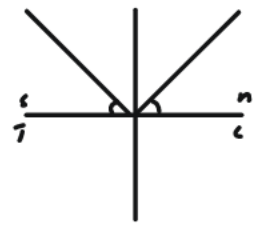
b) $0 \leq 3x - 15 \leq 3(180) - 15$

$0 \leq 3x - 15 \leq 525$

$\sin(3x-15) = \frac{1}{\sqrt{2}}$

$3x - 15 = 45, 135, 405, 495,$

$x = 20, 50, 140, 170$



7) $0 \leq \theta + 30 \leq 390$

$5\sin(\theta+30) = 4$

$\sin(\theta+30) = \frac{4}{5}$

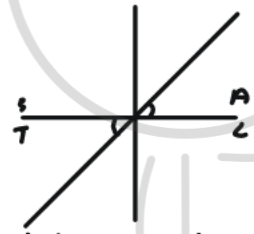
$\theta + 30 = 53.1, 126.9$

$\theta = 23.1, 96.9$

8) $5(-90) - 20 \leq 5x - 20 \leq 5(90) - 20$

$-470 \leq 5x - 20 \leq 430$

$\tan(5x-20) = \frac{5}{2}$



$5x - 20 = -291.8, -111.8, 68.2, 248.2, 428.2$

$x = -54.4, -18.4, 17.6, 53.6, 89.6$

BF MATHS

9) a) $\frac{\sin(2x)}{\cos(2x)} = 4\sin(2x)$

$\sin(2x) = 4\sin(2x)\cos(2x)$

$\sin(2x) - 4\sin(2x)\cos(2x) = 0$

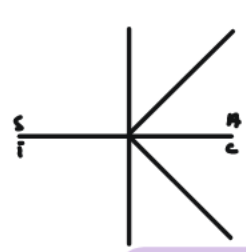
b) $0 \leq 2x \leq 360$

$\tan(2x) = 4\sin(2x)$

$(1 - 4\cos 2x)(\sin 2x) = 0$

$1 - 4\cos 2x = 0$

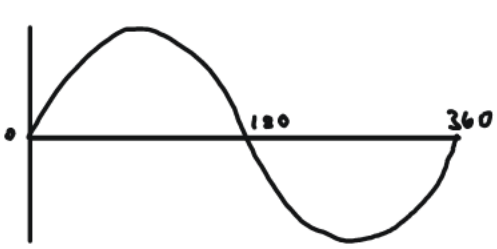
$\cos 2x = \frac{1}{4}$



$2x = 75.5, 284.5$ $x = 37.8\ 142.2$

$\sin 2x = 0$

$2x = 0, 180, 360$



$x = 0, 90, 180$

76) $\tan^2\frac{\theta}{2} = 16$, $0 \leq \theta \leq 360° \Rightarrow 0° \leq \frac{\theta}{2} \leq 180°$

$\left(\tan\frac{\theta}{2}\right)^2 = 16$

$\tan\frac{\theta}{2} = 4$ or $\tan\frac{\theta}{2} = -4$

$\tan^{-1}(4) = 75.96°$

$\frac{\theta}{2} = 75.96°$ (solution of 4) or $180° - 75.96 = 104.04°$ (solution of -4)

$\theta = 151.92°$ or $208.08°$