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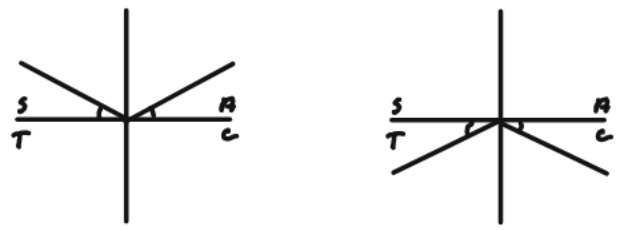
10.6 Equations and identities

1. a) $9\sin^2\theta = 1$

$9\sin^2\theta - 1 = 0$

$\sin\theta = \frac{1}{3}$ $\sin\theta = -\frac{1}{3}$

$\theta = 19.5$ $\theta = -19.5$



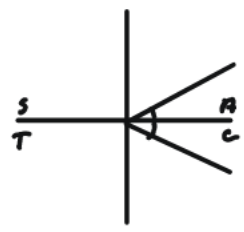
$\theta = 19.5, 161$ $\theta = 199, 341$

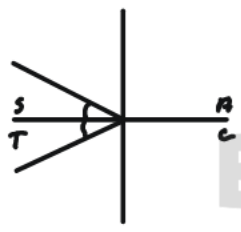
$\theta = 19.5, 161, 199, 341$

b) $4\cos^2\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$ $\cos\theta = -\frac{1}{2}$

$\theta = 60$ $\theta = 120$





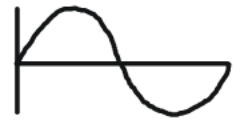
$\theta = 60, 300$ $\theta = 120, 240$

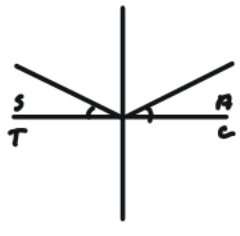
$\theta = 60, 120, 240, 300$

c) $5\sin^2\theta - \sin\theta = 0$

$\sin\theta = \frac{1}{5}$ $\sin\theta = 0$

$\theta = 11.5$





$\theta = 0, 180, 360$

$\theta = 11.5, 168$

$\theta = 0, 11.5, 168, 180, 360$

d) $\tan^2\theta + \tan\theta - 12 = 0$

$\tan\theta = 3$ $\tan\theta = -4$

$\theta = 71.6$ $\theta = -75.$

$\theta = 71.6, 105, 252, 285$

f) $0 \leq 3\theta \leq 3(360)$

$0 \leq 3\theta \leq 1080$

$\tan^2 3\theta = 4$

$\tan^2 3\theta - 4 = 0$

$\tan = 2$ $\tan = -2$

$\tan^{-1}(2)$

$3\theta = 63.4, 243.4, 423.4, 603.4,$

$783.4, 963.4$

$\tan^{-1}(-2)$

$3\theta = 116.5, 296.5, 476.5, 656.5$

$836.5, 1016.5$

$\theta = 21.1, 38.9, 81.1, 98.9, 141, 159$

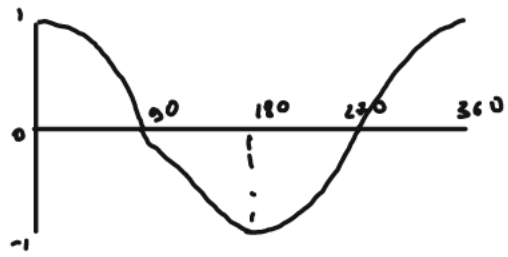
$201, 219, 261, 279, 321, 339.$

2) a) $0 \leq 3\theta \leq 1080$

$\cos^2 3\theta = 1$

$\cos^2 3\theta - 1 = 0$

$\cos 3\theta = 1$ or -1



$3\theta = 0$ or 180

$3\theta = 0, 180, 360, 540, 720, 900, 180$

$\theta = 0, 60, 120, 180, 240, 300, 360$

b) $\tan^2\theta = 5\tan\theta$

$\tan^2\theta - 5\tan\theta = 0$

$\tan\theta = 5$ or 0

$\theta = 78.7$ or 0

$\theta = 0, 78.7, 180, 259, 360$

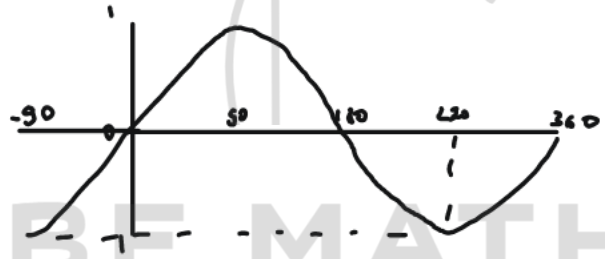
c) $0 \leq \theta \leq 180$

$\sin^2\theta - 3\sin - 4 = 0$

$\sin\theta = 4^x$ or -1

$\rightarrow$ out of range

$\theta = -90$



$\theta = 270$

d) $0+30 \leq \theta+30 \leq 360+30$

$30 \leq \theta+30 \leq 390$

$4\sin(\theta+30) = \tan(\theta+30)$

÷ sin

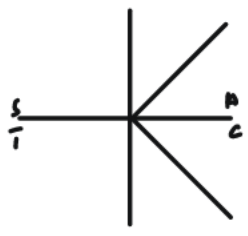
$4\sin(\theta+30) = \frac{\sin(\theta+30)}{\text{cosine}(\theta+30)}$

÷ sin when sin≠0

$4 = \frac{1}{\cos(\theta+30)}$

$4\cos(\theta+30) = 1$

$\cos(\theta+30) = \frac{1}{4}$



$\theta+30 = 75.5, 284.$ $\theta = 45.5, 254.5$

$\sin(\theta+30) = 0$

$\theta+30 = 0, 180, 360$

$\theta = 150, 330$

3) a) $2\cos^2 x - \sin^2 x - 4\cos x + 2 = 0$

$2\cos^2 x - (1 - \cos^2) - 4\cos x + 2 = 0$

$2\cos^2 - 1 + \cos^2 - 4\cos x + 2 = 0$

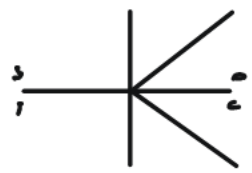
$3\cos^2 x - 4\cos x + 1 = 0$

b) $3\cos^2 x - 4\cos x + 1 = 0$

$\cos x = 1$ or $\frac{1}{3}$

$x = 0,\ 70,\ 5$

$x = 0, 70.5, 289.5, 360$



4) a) $5\cos^2\theta = 6(1 - \sin\theta)$

$5(1 - \sin^2) = 6 - 6\sin\theta$

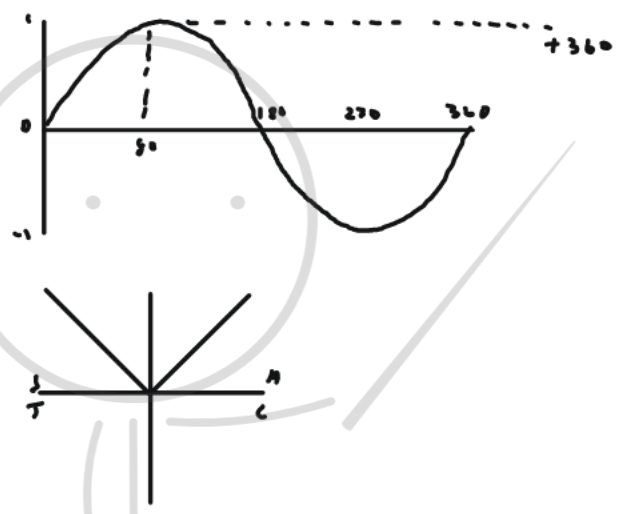
$5 - 5\sin^2 = 6 - 6\sin\theta$

$0 = 5\sin\theta - 6\sin\theta +$

$\sin\theta = 1$ or $\frac{1}{5}$

$\theta = 90$ or 11.5

$\theta = 11.5, 90, 168.5$



b) $8 - 7\cos\theta = 6\sin^2\theta$

$8 - 7\cos\theta = 6(1 - \cos^2\theta)$

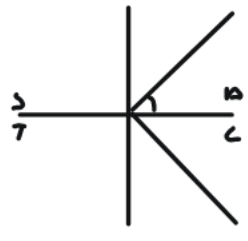
$8 - 7\cos\theta = 6 - 6\cos^2\theta$

$6\cos^2\theta - 7\cos\theta + 2 = 0$

$\cos\theta = \frac{2}{3}$ or $\frac{1}{2}$

$\theta = 48.2$ or 60

$\theta = 48.2, 60, 300, 311.8$



c) $3\tan\theta = 2\cos\theta$

$3\left(\frac{\sin\theta}{\cos\theta}\right) = 2\cos\theta$

$\frac{3\sin\theta}{\cos\theta} - 2\cos\theta = 0$

$$\frac{3\sin\theta}{\cos\theta} - 2\cos\theta = 0$$

$$\frac{3\sin\theta}{\cos\theta} \times \frac{2\cos\theta}{1} = 0$$

$$\frac{3\sin\theta - 2\cos^2\theta}{\cos\theta} = 0 \longrightarrow$$

If the quotient equals 0, it means that the division of the numerator by the denominator results in zero.

The only way this can happen is if the numerator itself is 0, because 0 divided by any non-zero number still equals 0. For exaple:

$\frac{0}{a} = 0$

where a is the denominator and must be non-zero.

If the numerator was any other number (positive or negative), then the result of the division would not be zero. For example:

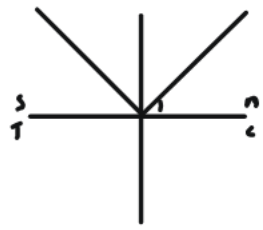
$\frac{5}{a} \neq 0$ or $\frac{-3}{a} \neq 0$

$$3\sin\theta - 2\cos^2\theta = 0$$

$$3\sin\theta - 2(1-\sin^2\theta) = 0$$

$$3\sin\theta - 2 + 2\sin^2\theta = 0$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad -2$$



$\theta = 30, 150^\circ$

5) a) $2\sin^2 x - 3\cos^2 x = 1$

$2\sin^2 x - 3(1-\sin x)^2 = 1$

$2\sin^2 x - 3 + 3\sin^2 x = 1$

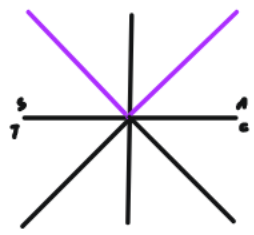
$5\sin^2 x - 3 = 1$

$5\sin^2 x = 4$

b) $5\sin^2 x - 4 = 0$

$\sin x = \frac{2\sqrt{5}}{5} \quad \text{or} \quad -\frac{2\sqrt{5}}{5}$ $\rightarrow -0.9$

$x = 63.4 \quad \text{or} \quad -63.4$



$X = 63.4, 116.6, 243.4, 296.6$

6) $2\cos^2 + \sin^2 x = \frac{10}{9}$

$2(1-\sin^2) + \sin^2 x = \frac{10}{9}$

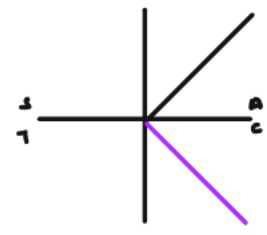
$2 - 2\sin^2 + \sin^2 x = \frac{10}{9}$

$-\sin^2 + \frac{8}{9} = 0$

$\sin x = \frac{2\sqrt{2}}{3} \quad \text{or} \quad -\frac{2\sqrt{2}}{3}$

$X = 70.5 \quad \text{or} \quad -70.5$

$X = 70.5, 109.5$



7) $2\cos^2 x + 2 = 7\sin x$

$2(1-\sin^2)+2=7\sin x$

$2-2\sin^2+2=7\sin x$

$-2\sin^2 x-7\sin x+4=0$

$\sin x = \frac{1}{2}$ or -4

$x = 30$ or no solutions

$x = 30^\circ, 150^\circ$

8) a) $\sin\theta \tan\theta = 2\cos\theta + 3$

$\frac{\sin\theta \sin\theta}{\cos\theta} = 2\cos\theta + 3$

$\sin^2\theta = 2\cos^2\theta + 3\cos\theta$

$3\cos^2 + 3\cos\theta - 1 = 0$

b) $3\cos^2 + 3\cos\theta - 1 = 0$

$\cos x = \frac{-3+\sqrt{21}}{6}$ or $\frac{-3-\sqrt{21}}{6}$

$x = 74.7$ or no solution

$x = 74.7, 285.3$

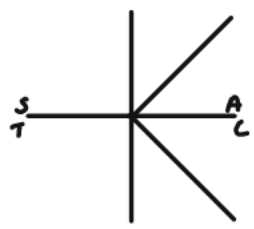
9) $15\sin^2 x + 8\cos x - 16 = 0$

$15(1-\cos^2)+8\cos x-16=0$

$15-15\cos^2+8\cos x-16=0$

$-15\cos^2+8\cos x-1=0$

$\cos x = \frac{1}{3}$ or $\frac{1}{5}$



$x = 70.5$ or 78.5

$x = 430.5, 438.5$