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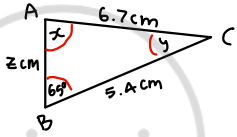
9.4 Solving triangle problems

1.

a) angle $x°$ → $\frac{\sin 65}{6.7} = \frac{\sin x}{5.4}$

$x = \sin^{-1}\frac{\sin 65}{6.7} \times 5.4$

$x = 46.9°$ (3 sf)



angle $y°$ → $180° - (65° + 46.9°)$

$y = 68.1°$ (3 sf)

length z cm → $z^2 = AC^2 + BC^2 - 2(AC)(BC)\cos y$

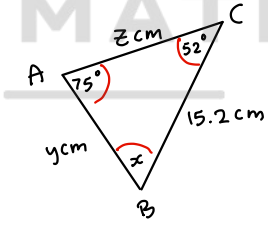
$z^2 = 6.7^2 + 5.4^2 - 2(6.7)(5.4)\cos 68.1$

$z = \sqrt{6.7^2 + 5.4^2 - 2(6.7)(5.4)\cos 68.1}$

$z = 6.86$cm (3 sf)

b) angle $x°$ ⟶ $180° - (75° + 52°)$

$x = 53.0°$ (3 sf)



length y cm ⟶ $\frac{15.2}{\sin 75} = \frac{y}{\sin 52}$

$y = \frac{15.2}{\sin 75} \times \sin 52$

$y = 12.4$cm (3 sf)

length z cm ⟶ $\frac{15.2}{\sin 75} = \frac{z}{\sin x}$ ⟶ $\frac{15.2}{\sin 75} = \frac{z}{\sin 53}$

$\frac{15.2}{\sin 75} \times \sin 53$

$z = 12.6$cm (3 sf)

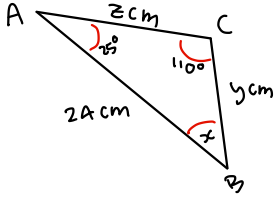
c) angle $x°$ ⟶ $180° - (110° + 25°)$

$x = 45.0°$ (3 sf)

length y cm ⟶ $\frac{24}{\sin 110} = \frac{y}{\sin 25}$

$y = \frac{24}{\sin 110} \times \sin 25$

$y = 10.8$cm (3 sf)

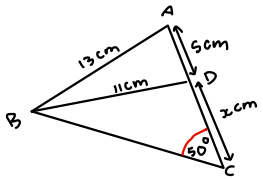


length z cm → $\frac{24}{\sin 110} = \frac{z}{\sin x}$ → $\frac{24}{\sin 110} = \frac{z}{\sin 45}$

$z = \frac{24}{\sin 110} \times \sin 45$

$z = 18.1$cm (3 sf)

2.



a) $\angle BAC \rightarrow \cos A = \frac{AB^2+AD^2-BD^2}{2(AB)(AD)}$

$\cos A = \frac{13^2+5^2-11^2}{2(13)(5)}$

$\cos^{-1}\frac{13^2+5^2-11^2}{2(13)(5)}$

$\cos A = 55.8^\circ$ (3 sf)

$BC \rightarrow \frac{13}{\sin 50} = \frac{BC}{\sin 55.8}$

$\frac{13}{\sin 50} \times \sin 55.8$

$BC = 14.0$ (3 sf)

$\angle BDC \rightarrow \frac{\sin 50}{11} = \frac{\sin D}{14.0}$

$\frac{\sin 50}{11} \times 14.0$

$\sin^{-1}\frac{\sin 50}{11} \times 14.0$

$\angle BDC = 77.8^\circ$ (3 sf)

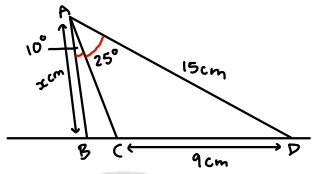
$\angle DBC = 180^\circ - (77.8^\circ + 50^\circ)$

$= 52.2^\circ$ (3 sf)

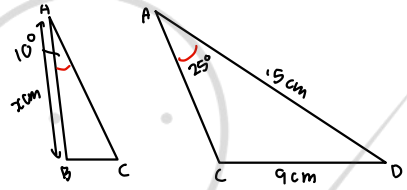
$x^2 = 14.0^2 \times 11^2 - 2(14.0)(11) \times \cos 52.2$

$x = \sqrt{14.0^2 \times 11^2 - 2(14.0)(11) \times \cos 52.2}$

$x = 11.3\text{cm}$ (3 sf)



b)



$\angle ACD = \frac{\sin 25}{9} = \frac{\sin C}{15}$

$\sin C = \sin^{-1}\frac{\sin 25}{9} \times 15$

$\sin C = 44.77816685^\circ$

$\angle ACD$ is obtuse

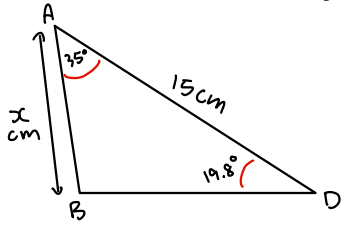
$\therefore 180 - 44.77\ldots$

$\angle ACD = 135.2218331^\circ$

$\therefore \angle ACB = 44.77\ldots$

$\angle ADC = 180 - (25 + 135.2\ldots)$

$= 19.77816685^\circ$



$\angle BAD = 35^\circ$

$\angle ACD = 180^\circ - (35 + 19.8\ldots)$

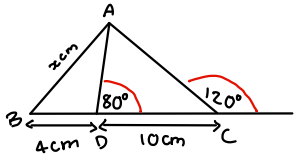
$= 125.2218331^\circ$

$x = \frac{15}{\sin 125\ldots} = \frac{x}{\sin 19.8\ldots}$

$x = \frac{15}{\sin 125} \times \sin 19.8\ldots$

$x = 6.21\text{cm}$ (3 sf)

2 c)



$\angle ACD \longrightarrow$ angles on a straight line = 180°
so $180° - 120° = 60°$

$\angle CAD \longrightarrow 180° - (80° + 60°)$
$= 40°$

$\angle ABD \longrightarrow$ angles on a straight line = 180°
so $180° - 80° = 100°$

$AD \longrightarrow \dfrac{10}{\sin 40} = \dfrac{AD}{\sin 60}$

$\dfrac{10}{\sin 40} \times \sin 60$

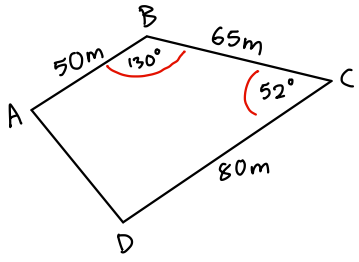
$= 13.5\,cm$ (3 sf)

$x^2 = 4^2 + 13.5^2 - 2(4)(13.5) \times \cos 100$

$x = \sqrt{4^2 + 13.5^2 - 2(4)(13.5) \times \cos 100}$

$x = 14.7\,cm$ (3 sf)

3.



$\triangle ABC \longrightarrow \frac{1}{2} \times 50 \times 65 \times \sin 130$

$= 1240 m^2$ (3 sf)

$AC \longrightarrow \sqrt{50^2 + 65^2 - 2(50)(65) \times \cos 130}$

$= 104 m$ (3 sf)

$\angle BCA \longrightarrow \frac{\sin 130}{104} = \frac{\sin x}{50}$

$\frac{\sin 130}{104} \times 50$

$\sin^{-1} \frac{\sin 130}{104} \times 50$

$= 21.5°$ (3sf)

$\angle ACD \longrightarrow 52 - 21.5$

$= 30.5$ (3sf)

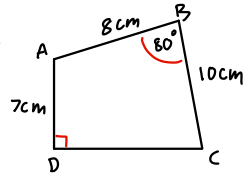
$\triangle ACD \longrightarrow \frac{1}{2} \times 104 \times 80 \times \sin 30.5$

$= 2120 m^2$ (3 sf)

Total area $= 1240 + 2120$

$= 3360 m^2$ (3sf)

4.



$\triangle ABC \longrightarrow AC^2 = 8^2 + 10^2 - 2(8)(10) \times \cos 80$

$AC = \sqrt{8^2 + 10^2 - 2(8)(10) \times \cos 80}$

$= 11.67117353\,cm$

$\text{area} = \frac{1}{2} \times 8 \times 10 \times \sin 80$

$= 39.39231012\,cm^2$

$\triangle ADC \longrightarrow DC^2 = AC^2 - AD^2$

$DC^2 = 11.7^2 - 7^2$

$DC = \sqrt{11.67\ldots^2 - 7^2}$

$= 9.338966301\,cm$

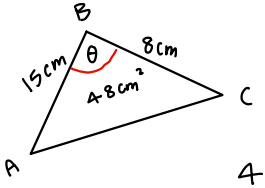
$\text{area} = \frac{1}{2} \times 9.338\ldots \times 7$

$= 32.68638205\,cm^2$

$\text{total area} = 39.39\ldots + 32.68\ldots$

$= 72.1\,cm^2\ (3sf)$

5.



a) $\sin\theta = \dfrac{48}{\frac{1}{2} \times 15 \times 8}$

$= \sin^{-1} \dfrac{48}{\frac{1}{2} \times 15 \times 8}$

$= 53.1^\circ\ (3sf)$

$\cos 53.1 = \frac{3}{5} \quad \text{or} \quad -\frac{3}{5}$

$\cos\theta = \pm\frac{3}{5}$

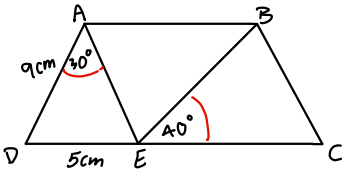
b) $\angle ABC$ is obtuse so $\cos\theta = -\frac{3}{5}$

$AC^2 = 15^2 + 8^2 - 2(15)(8) \times \left(-\frac{3}{5}\right)$

$AC = \sqrt{15^2 + 8^2 - 2(15)(8) \times \left(-\frac{3}{5}\right)}$

$= \sqrt{433}\ cm$

6.



$$\angle AED = \frac{\sin 30}{5} = \frac{\sin E}{9}$$

$$\sin E = \sin^{-1}\frac{\sin 30}{5} \times 9$$

$$= 64.15800724^\circ$$

$$\angle ADE = 180^\circ - (30^\circ + 64.15\ldots^\circ)$$

$$= 85.8419327^\circ$$

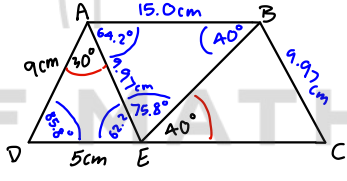
$$\angle AEB = 180^\circ - (64.15\ldots^\circ + 40^\circ)$$

$$= 75.84199276^\circ$$

ABCE is a parallelogram

$\therefore \angle BEC = \angle ABE$

$\therefore \angle ABE = 40^\circ$



$$\angle BAE = 180^\circ - (75.84\ldots^\circ + 40^\circ)$$

$$= 64.15800724^\circ$$

$$\frac{5}{\sin 30} = \frac{AE}{\sin 85.8\ldots}$$

$$AE = \frac{5}{\sin 30} \times \sin 85.8\ldots$$

$$AE = 9.973678106\ \text{cm}$$

$$AB = \frac{9.97\ldots}{\sin 40} = \frac{AB}{\sin 75.84\ldots}$$

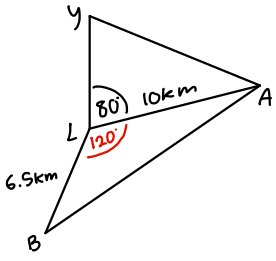
$$AB = \frac{9.97\ldots}{\sin 40} \times \sin 75.84\ldots$$

$$= 15.04497951\ \text{cm}$$

$$\triangle ABE = \frac{1}{2} \times 9.97\ldots \times 15.0\ldots \times \sin 64.158\ldots$$

$$= 67.5\ \text{cm}^2\ (3\ \text{sf})$$

7.



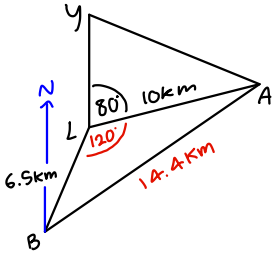
$\angle BLA = 200° - 80°$
$= 120°$

a) $AB^2 = 6.5^2 + 10^2 - 2(6.5)(10) \times \cos 120$

$AB = \sqrt{6.5^2 + 10^2 - 2(6.5)(10) \times \cos 120}$

$AB = 14.4\text{km}$ (3sf)

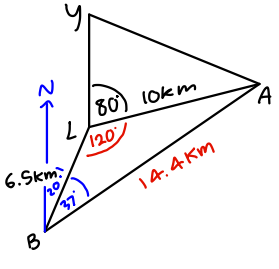
b)



$\angle LBA = \frac{\sin 120}{14.4} = \frac{\sin B}{10}$

$\sin B = \sin^{-1} \frac{\sin 120}{14.4} \times 10$

$\angle LBA = 37°$



$\angle YLB = 360° - (120° + 80°)$
$= 160°$

$\angle NBL = 180° - 160°$
$= 20°$ (co-interior angles)

Bearing of A from B $= 20° + 37° = 57°$
$= 057°$ (nearest degree)