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7.4 - Mathematical Proof.

① $(x - 4)(x+6)(2x+3) \equiv 2x^3 + 7x^2 - 42x - 72$

LHS -

$(x-4)(x+6)(2x+3)$

$(x^2 + 6x - 4x - 24)(2x+3)$

$(x^2 + 2x - 24)(2x+3)$

$2x^3 + 3x^2 + 4x^2 + 6x - 48x - 72$

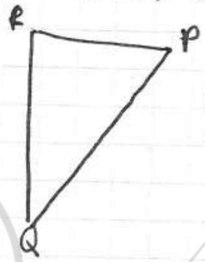
$2x^3 + 7x^2 - 42x - 7$

LHS $\equiv$ RHS

$2x^3 + 7x^2 - 42x - 7 \equiv 2x^3 + 7x^2 - 42x - 72$

② P (10, 14), Q (-6, 2) & R (12, 8)

Use Pythagoras theorem -



$a^2 + b^2 = c^2$

- $PR^2 = 2^2 + 6^2$
  $= 40.$
- $PQ^2 = 16^2 + 12^2$
  $= 400$
- $QR^2 = 18^2 + 6^2$
  $= 360$

$QR^2 + PR^2 = PQ^2$ as:

$360 + 40 = 400.$

so ∠PQR is a right-angled triangle.

③ Tangent intersects curve only once so -

$kx = 4x^2 - 5x + 4.$

$4x^2 - 5x - kx + 4 = 0$

$4x^2 - x(5+k) + 4 = 0$

$b^2 - 4ac = 0$

$a = 4,\ b = 5+k,\ c = 4.$

$(5+k)^2 - 4(4 \times 4) = 0$

$k^2 + 10k + 25 - 4(16) = 0$

$k^2 + 10k + 25 - 64 = 0$

$k^2 + 10k - 39 = 0.$

$k^2 + 13k - 3k - 39 = 0$

$k(k+13) - 3(k+13) = 0$

$(k+13)(k-3) = 0$

$(k+13)(k-3) = 0$

$k = 3 \ / \ -13.$

$y = kx$

$y = 3x \ / \ y = -13x$

lines with these 2 equations are tangents to the curve!

(4) if A, B, C are collinear, their line segments' gradients are equal.

gradient of BC :

$B \to (2,1)$ $C \to (14, -5)$

$$\frac{-5-1}{14-2} = -\frac{1}{2}$$

gradient of AB :

$A \to (-2,3)$ $B \to (2,1)$

$$\frac{1-3}{2+2} = -\frac{1}{2}.$$

gradient of both $= -\frac{1}{2}$ with B as a common point.

Thus A, B and C are collinear.

(5) $x^2 - 6x + 10 > 0$

$(x-3)^2 - 9 + 10 > 0$

$(x-3)^2 + 1 > 0$

$(x-3)^2 + 1 \geq 1$

as this $(x-3)^2 \geq 0$ for all values of x.

(6) $(x+y)^2 - (x-y)^2 \equiv 4xy$

LHS

$(x+y)^2 - (x-y)^2$

$x^2 + 2xy + y^2 - (x^2 - 2xy + y^2)$

$x^2 + 2xy + y^2 - x^2 + 2xy - y^2$

$= 4xy$

RHS

$4xy$

LHS $\equiv$ RHS

$4xy \equiv 4xy$.

(7) let odd number be $2n+1$. next (consecutive) number $= 2n+3$

$(2n+3)^2 - (2n+1)^2$

$= 4n^2 + 12n + 9 - (4n^2 + 4n + 1)$

$= 4n^2 + 12n + 9 - 4n^2 - 4n - 1$

$= 8n + 8$.

$8n + 8$

$8(n+1)$, always a multiple of 8.

(8) let odd number be $2n+1$

cube - square

$(2n+1)^3 - (2n+1)^2$

$= 8n^3 + 12n^2 + 6n + 1 - (4n^2 + 4n + 1)$

$= 8n^3 + 8n^2 + 2n$.

$2(4n^3 + 4n^2 + n)$

always even as it is always multiplied by 2.

9) $x^2 + (k-3)x + (3-2k) = 0$

no real roots $\Rightarrow$ $b^2 - 4ac < 0$

$(k-3)^2 - 4(1)(3-2k) < 0$

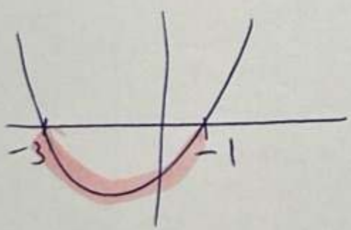
$k^2 - 6k + 9 - 4(3-2k) < 0$

$k^2 - 6k + 9 - 12 + 8k < 0$

$k^2 + 2k - 3 < 0$

$(k+3)(k-1) < 0$

$k = -3$ or $k = 1$



$$\underline{\underline{-3 < k < 1}}$$