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1.3 - The Factor Theorem

(1) a) $(x+1)$ is a factor of $2x^3+7x^2-5$ (given)

$x+1=0$ $\quad 2(-1)^3+7(-1)^2-5=0$

$x=-1$ $\quad f(-1)=0$

b) $(x+2)=0$ $\quad f(-2)$ → $(-2)^3+4(-2)^2+3(-2)-2=0$

$x=-2$ $\quad f(-2)=0$

c) $x-3=0$ $\quad f(3)=0$

$x=3$ $\quad 2(3)^3-3(3)^2-7(3)-6=0$

d) $x-4=0$ $\quad f(4)=0$

$x=4$ $\quad (4)^4-3(4)^3-15(4)-4=0$

(2) a) $(x-2)$, $2x^3+x^2-13x+6$

$x-2=0$ $\quad f(2)=0$

$x=2$

$$x-2 \;\big|\; 2x^3+x^2-13x+6 \qquad 2x^2+5x-3$$

$-2x^3-4x^2$

$5x^2-13x$

$-5x^2-10x$

$-3x+6$

$-3x+6$

0

$(x-2)(2x^2+5x-3)$ ↓ (factorise)

Ans - $f(2)=0$, $(x-2)(2x+1)(x+3)$

b) $(x+3)$, $2x^3+17x^2+38x+15$

$x+3=0$ $\quad f(-3)=0$

$x=-3$

$$x+3 \;\big|\; 2x^3+17x^2+38x+15 \qquad 2x^2+11x+3$$

$-2x^3+6x^2$

$11x^2+38x$

$-11x^2+33x$

$5x+15$

$-5x+15$

0

$(x+3)(2x^2+11x+3)$

$=(x+3)(x+5)(2x+1)$

Ans - $f(-3)=0$, $(x+3)(x+5)(2x+1)$

BT MATHS

2c) ~~f(x)~~ $(x-1)$, $6x^3 - x^2 - 11x + 6$

$x - 1 = 0$ $\quad f(1) = 0$

$x = 1$

$$x-1 \;\big|\; 6x^3 - x^2 - 11x + 6$$

$6x^2 + 5x - 6$

$-\,6x^3 - 6x^2$

$5x^2 - 11x$

$-\,5x^2 - 5x$

$-6x + 6$

$-\,-6x + 6$

0

$(x-1)(6x^2 + 5x - 6)$

$(x-1)(3x-2)(2x+3)$

Ans -

$f(1) = 0 \cdot (x-1)(3x-2)(2x+3)$

d) $(x+4)$, $15x^3 + 61x^2 - 2x - 24$

$x + 4 = 0$ $\quad f(-4) = 0$

$x = -4$

$$x+4 \;\big|\; 15x^3 + 61x^2 - 2x - 24$$

$15x^2 + x - 6$

$-\,15x^3 + 60x^2$

$x^2 - 2x$

$-\,x^2 + 4x$

$-6x - 24$

$+\,-6x - 24$

0

$(x+4)(15x^2 + x - 6)$

$(x+4)(5x-3)(3x+2)$

Ans -

$f(-4) = 0 \; (x+4)(5x-3)(3x+2)$

(3) a) $x^3 + 2x^2 - 21x + 18$

$(1)^3 + 2(1)^2 - 21(1) + 18$

$= 0$

$f(1) = 0$ $\quad x = 1$

$x - 1 = 0$

Ans -

$(x-1)(x^2 + 3x - 18)$

$(x-1)(x-3)(x+6)$

$$x-1 \;\big|\; x^3 + 2x^2 - 21x + 18$$

$x^2 + 3x - 18$

$-\,x^3 - x^2$

$3x - 21x$

$-\,3x - 3x$

$-18x + 18$

$-\,-18x + 18$

0

b) $2x^3 + 13x^2 + 13x - 10$

$2(-2)^3 + 13(-2)^2 + 13(-2) - 10$

$= 0$

$x = -2$ $\quad x + 2 = 0$

$$x+2 \;\big|\; 2x^3 + 13x^2 + 13x - 10$$

$2x^2 + 9x - 5$

$-\,2x^3 + 4x^2$

$9x^2 + 13x$

$-\,9x^2 + 18x$

$-5x - 10$

$-\,-5x - 10$

0

$(x+2)(2x^2 + 9x - 5)$

$= (x+2)(x+5)(2x-1)$

BF MATHS

3c) $3x^3 + 2x^2 - 41x - 60$

$3(-3)^3 + 2(-3)^2 - 41(-3) - 60 = 0$.

$x = -3 \quad x + 3 = 0$

$$\begin{array}{r|l} & 3x^2 - 7x - 20 \\ \hline x+3 & 3x^3 + 2x^2 - 41x - 60 \\ & -3x^3 + 9x^2 \\ \hline & \quad -7x^2 - 41x \\ & \quad -7x^2 - 21x \\ \hline & \quad\quad -20x - 60 \\ & \quad\quad -20x - 60 \\ \hline & \quad\quad\quad 0 \end{array}$$

$(x+3)(3x^2 - 7x - 20)$

$(x+3)(x-4)(3x+5)$

Ans.

(4) a) i) $2x^3 - 11x^2 + 5x + 18$

$2(2)^3 - 11(2)^2 + 5(2) + 18 = 0$

$x = 2$

$x - 2 = 0$

$$\begin{array}{r|l} & 2x^2 - 7x - 9 \\ \hline x-2 & 2x^3 - 11x^2 + 5x + 18 \\ & -2x^3 + 4x^2 \\ \hline & \quad -7x^2 + 5x \\ & \quad -7x^2 + 14x \\ \hline & \quad\quad -9x + 18 \\ & \quad\quad -9x + 18 \\ \hline & \quad\quad\quad 0 \end{array}$$

$(x-2)(2x^2 - 7x - 9)$

Ans -

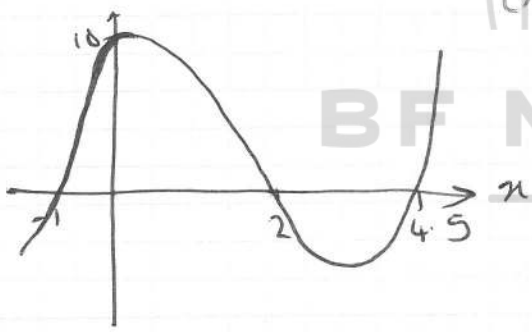
$(x-2)(x+1)(2x-9)$

$x = 2 \;/-1/\; 4.5$

$2x^3 - 11x^2 + 5x + (18)$

↳ y intercept

ii)



b) i) $2x^3 - 3x^2 - 39x + 20$

$[2(5)^3 - 3(5)^2 - 39(5) + 20 = 0$

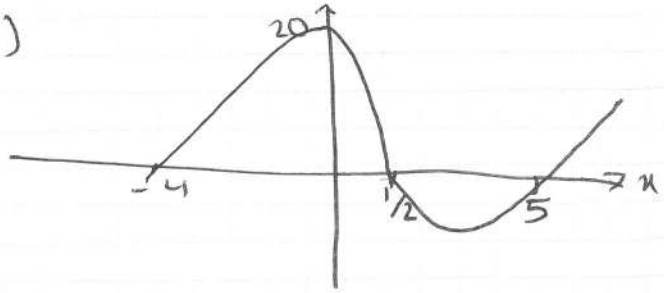
$x = 5 \qquad x - 5 = 0$

$$\begin{array}{r|l} & 2x^2 + 7x - 4 \\ \hline x-5 & 2x^3 - 3x^2 - 39x + 20 \\ & -2x^3 - 10x^2 \\ \hline & \quad 7x^2 - 39x \\ & \quad -7x^2 - 35x \\ \hline & \quad\quad -4x + 20 \\ & \quad\quad -4x + 20 \\ \hline & \quad\quad\quad 0 \end{array}$$

$(x-5)(2x^2 + 7x - 4)$

$(x-5)(x+4)(2x-1)$

ii)



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4c) i) $6x^3 + 37x^2 + 50x - 21$ ← y intercept

$x = -3$

$6(-3)^3 + 37(-3)^2 + 50(-3) - 21 = 0$

$x = -3$ as $x + 3 = 0$

$6x^2 + 19x - 7$

$x+3 \;\overline{)\; 6x^3 + 37x^2 + 50x - 21}$

$-(6x^3 + 18x^2)$

$19x^2 + 50x$

$-(19x^2 + 57x)$

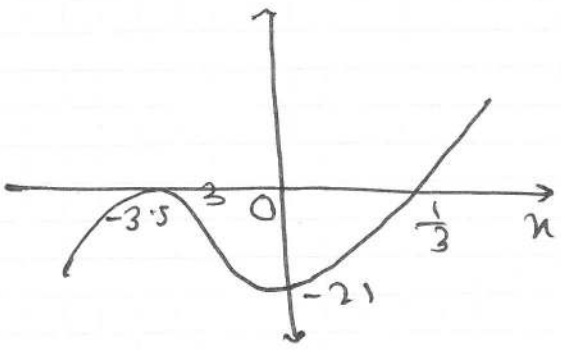
$-7x - 21$

$-(-7x - 21)$

0

$6x^2 + 19x - 7 \rightarrow (x+3)(3x-1)(2x+7)$

ii) $x = -3, \frac{1}{3}, -3.5$



(5)

$6x^2 + x - 12$

$x-3 \;\overline{)\; 6x^3 - 17x^2 + 15x + 36}$

$-(6x^3 - 18x^2)$

$x^2 + 15x$

$-(x^2 - 3x)$

$-12x + 36$

$-(-12x + 36)$

0

$(x-3)(6x^2 + x - 12)$

$(x-3)(3x-4)(2x+3)$

$x = 3, \; x = \frac{4}{3}, \; -\frac{3}{2}$.

(6) b) $f(x) = 9x^3 + 24x^2 - 44x + 16$.

$9x^2 - 12x + 4$

$x+4 \;\overline{)\; 9x^3 + 24x^2 - 44x + 16}$

$-(9x^3 + 36x^2)$

$-12x^2 - 44x$

$-(-12x^2 - 48x)$

$4x + 16$

$-(4x + 16)$

0

$(x+4)(9x^2 - 12x + 4)$

b) $(x+4)(9x^2 - 12x + 4)$

$(x+4)(3x-2)^2$

a) $x + 4 = 0$

$x = -4$

$9(-4)^3 + 24(-4)^2 - 44x + 16 = 0$ (-4)

$f(-4) = 0$

BF MATHS

(7) $f(x) = 2x^3 - 3x^2 - 5x + 6$

$x \mp 1 = 0 \quad x = +1$

$2(-1)^3 - 3(-1)^2 - 5(-1) + 6 = 0$

$$\begin{array}{r|l} & 2x^2 - x - 6 \\ \hline x-1 & 2x^3 - 3x^2 - 5x + 6 \\ & -2x^3 - 2x^2 \\ \hline & \quad -x^2 - 5x \\ & -(-x^2 + x) \\ \hline & \quad -6x + 6 \\ & -(-6x + 6) \\ \hline & \quad 0 \end{array}$$

$(x-1)(2x^2 - x - 6)$

$2x^2 - x - 6$

$2x^2 - 4x + 3x - 6$

$2x(x-2) + 3(x-2)$

$(2x+3)(x-2)$

$(x-1)(2x+3)(x-2)$

$x = 1, \; x = \frac{-3}{2}, \; 2.$

(8) a) $x + 1 = 0$

$x = -1 \rightarrow$ substitute into x value.

$(-1)^3 + 2(-1)^{\frac{2}{3}} - 19(-1) + k = 0$

$20 + k = 0$

$k = -20.$

b) $x^3 + 2x^2 - 19x - 20.$

$$\begin{array}{r|l} & x^2 + x - 20 \\ \hline x+1 & x^3 + 2x^2 - 19x - 20 \\ & -x^3 + x^2 \\ \hline & \quad x^2 - 19x \\ & -(x^2 + x) \\ \hline & \quad -20x + k - 20 \\ & -(-20x - 20) \\ \hline & \quad 0 \end{array}$$

$(x+1)(x^2 - x - 20)$

$x^2 - 5x + 4x - 20 = 0$

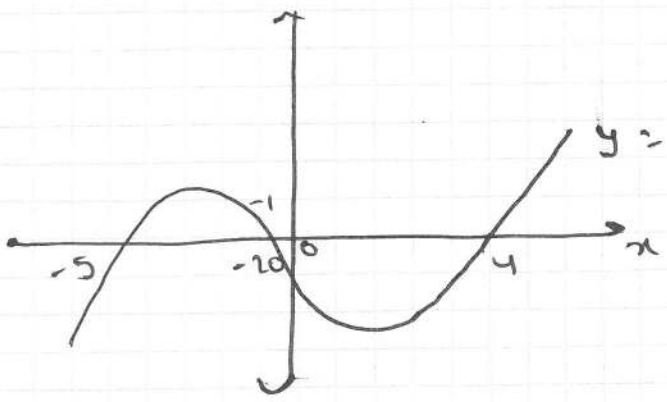
$x(x-5) + 4(x-5) = 0$

$(x+4)(x-5) = 0$

$x = -4, \; 5$

$x + 1 = 0, \; x = -1.$

c) $x = -1, -4, 5.$ $\quad (x+1)(x+4)(x-5) = 0$



BF MATHS

(9) a) $p(x) = 25x^3 + 55x^2 - 56x + 12$

$x + 3 = 0 \quad x = -3$

$25(-3)^3 + 55(-3)^2 - 56(-3) + 12$

$= 0$

$f(-3) = 0$,

b)

$$\begin{array}{r|l} & 25x^2 - 20x + 4 \\ \hline x+3 & 25x^3 + 55x^2 - 56x + 12 \\ & -25x^3 + 75x^2 \\ \hline & \quad -20x^2 - 56x \\ & -(-20x^2 - 60x) \\ \hline & \quad 4x + 12 \\ & -(4x + 12) \\ \hline & \quad 0 \end{array}$$

$(x+3)(25x^2 - 20x + 4)$

$(x+3)(25x^2 - 10x - 10x + 4)$

$(x+3)(5x(5x-2) - 2(5x-2))$

c) $(x+3)(5x-2)(5x-2)$

$(x+3)(5x-2)^2$

$x = -3, \quad x = \frac{2}{5}$.

BF MATHS