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6.3 Intersection of straight lines and circles.

1)a) $y=0$

$(x-2)^2+(0-4)^2=52$

$(x-2)^2+(-4)^2=52$

$(x-2)^2=52-16=36$

$x-2=\sqrt{36}\rightarrow 6$

$x=6+2$

$x=8$ $\quad (8,0)$

b) $x=0$

$(0-2)^2+(y-4)^2=52$

$(-2)^2+(y-4)^2=52$

$4+(y-4)^2=52$

$(y-4)^2=48$

$y-4=\sqrt{48}+4$

$y=-4$ $\quad (0,-4)$

2)a) $2x+y=7$

$y=-2x+7$

$(x-1)^2+(-2x+7)^2=50$

$x^2-2x+1+4x^2-28x+49=50$

$5x^2-30x+50=50$

$5x^2-30x=0$

$x=0$ or $x=6$

$y=-2(0)+7$ $\quad y=-2(6)+7$

$y=7$ $\quad y=-5$

$(0,7)$ $\quad (6,-5)$

3)a) $2x+y-15=0$

$y=-2x+15$

$(x-1)^2+(-2x+15-4)=50$

$(x-1)^2+(-2x+11)^2=50$

$x^2-2x+1+4x^2-44x+121=150$

$5x^2-46x+121=150 \rightarrow 5x^2-46x-29=0 \rightarrow x^2-8x-16=0$

b^2-4ac

$(-8)^2-4(1)(16)=0$

meets at one point.

b) xxx

b) $x^2 - 8x + 16 = 0$

$x = 4$

$2(4) + y - 15 = 0$

$8 + y = 15$

$y = 7$

$(4, 7)$

4) $y = 2x + 6$

$(x-5)^2 + (2x+6-2)^2 = 30$

$(x-5)^2 + (2x+4)^2 = 30$

$x^2 - 10x + 25 + 4x^2 + 16x + 16 = 30$

$= 5x^2 + 6x + 41 = 30$

$= 5x^2 + 6x + 11 = 0$

$a = 5$
$b = 6$
$c = 11$

$b^2 - 4ac$

$(6)^2 - 4(5)(11)$

$= 36 - 220 = -184$

$b^2 - 4ac < 0$ does not meet the circle.

5) $x^2 + y^2 = 100$

Centre is at origin and radius is at 10.

b) $3x + y - 10 = 0$

$y = -3x + 10$

$x^2 + (-3x + 10)^2 = 100$

$x^2 + 9x^2 - 60x + 100 = 100$

$10x^2 - 60x = 0$

$x = 6$ or $x = 0$

$3(6) + y = 10$ $\quad$ $3(0) + y = 10$

$18 + y = 10$ $\quad$ $0 + y = 10$

$y = -8$ $\quad$ $y = 10$

$(6, -8)$ $\quad$ $(0, 10)$

6)a) $y = x + k$

$x^2 + (x+k)^2 - 6x - 23 = 0$

$x^2 + x^2 + 2kx + k^2 - 6x - 23 = 0$

$2x^2 + 2kx + 6x + k^2 - 23 = 0$

$2x^2 + 2(k-3)x + k^2 - 23 = 0$

b) $b^2 - 4ac = 0$ $\quad a = 2$

$(2k-6)^2 - 4(2)(k^2-23)$ $\quad b = 2k - 6$

$4k^2 - 24k + 36 - 8k^2 + 184$ $\quad c = k^2 - 23$

$\frac{-4k^2 - 24k + 220 = 0}{-4}$

$k^2 + 6k - 55 = 0$

$k = 5$ or $k = -11$

c) The line is a tangent to the circle.

7) a) $x + 11y = 119$

$x = -11y + 119$

$(-11y + 119 - 3)^2 + (y-5)^2 = 61$

$(-11y + 116)^2 + (y-5)^2 = 61$

$121y^2 - 2552y + 13456 + y^2 - 10y + 25 = 61$

$122y^2 - 2562y + 13481 = 61$

$122y^2 - 2562y + 13420 = 0$

$y^2 - 21y + 110 = 0$

$y = 11$ $\quad y = 10$

$x + 11(11) = 119$ $\quad x + 11(10) = 119$

$x = 119 - 121 = -2$ $\quad x = 119 - 110 = 9$

$(-2, 11)$ $\quad (9, 10)$

b) gradient $l = \frac{\Delta y}{\Delta x} = \frac{11-10}{-2-9} = \frac{1}{-11}$

$y - 10.5 = 11(x - 3.5)$

$y = 11x - 28$

l_2 $m = 11$

midpoint $= \left(\frac{-2+9}{2}, \frac{11+10}{2}\right) = \frac{7}{2}, \frac{21}{2}$ $\quad (3.5, 10.5)$

7)c) Centre: $(3,5)$

$x = 3$

$y = 11(3) - 28$

$y = 33 - 28$

$y = 5$

Passes through centre.

8) $x^2 + y^2 - 10x + 8y + 25 = 0$

$x^2 - 10x + y^2 + 8y = -25$

$(x-5)^2 - 25 + (y+4)^2 - 16 = -25$

$(x-5)^2 + (y+4)^2 = 16$

Centre: $(5, -4)$ radius: $\sqrt{16} = 4$

$y = kx$

$x^2 + (kx)^2 - 10x + 8(kx) + 25 = 0$

$x^2 + k^2x^2 - 10x + 8kx + 25 = 0$

$(1+k^2)x^2 + (8k-10)x + 25 = 0$

$a \qquad b \qquad c$

$b^2 - 4ac > 0$

$(8k-10)^2 - 4(1+k^2)(25) > 0$

$-36k^2 - 160k > 0$

$36k^2 + 160k < 0$

$4k(9k + 40) < 0$

$k > 0 \qquad k = -40/9$

$-40/9 < k < 0$

9) a) $(4, 7)$

$x + 3y = k$

$4 + 3(7) = k$

$4 + 21 = 25$

$k = 25$

9)b) (4,7)

$(4-p)^2 - (7-4)^2 = 45$

$(4-p)^2 - (3)^2 = 45$

$(4-p)^2 + 9 = 45$

$(4-p)^2 = 36$

$4-p = \sqrt{36} = \pm 6$

$4-p = 6$ $\qquad$ $4-p = -6$

$p = -2$ $\qquad$ $p = 10$

p is 10 or −2