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7.5 - Methods of proof

① a) $n = 5 \rightarrow 5^2 + 5 + 17 = 47$

$n = 7 \rightarrow 7^2 + 7 + 17 = 73$

$n = 9 \rightarrow 9^2 + 9 + 17 = 107$

$n = 6 \rightarrow 6^2 + 6 + 17 = 59$

$n = 8 \rightarrow 8^2 + 8 + 17 = 89$

$n = 10 \rightarrow 10^2 + 10 + 17 = 127$

Proof by exhaustion

$5 \leq n \leq 10$

$f(n) = \text{prime}$.

b) $n = 17 \rightarrow 17^2 + 17 + 17$

$= \underbrace{17}(17 + 1 + 1)$

divisible by 17 so it is not a prime number

(Proof by counter example)

② 2 & 3 = both prime numbers

$3 - 2 = \underline{1}$

an odd number not even

so no, difference of 2 prime numbers is not always even.

③ a) $(1+2x)^2 - 4x^2 - 1 = 4x > 0$ (for all + values of x)

so $(1+2x)^2 - 4x^2 - 1 > 0 \rightarrow (1+2x)^2 > 1 + 4x^2$.

b) if $x = -1$, $(1+2x)^2 = 1$, and $1 + 4x^2 = 5$

(Proof by counter example) so no $(1+2x)^2 > 1+4x^2$ is not true for all values of x.

4) a) cancelling a factor of x is not valid when $x = 0$

b) $x = 0$.

⑤ when $x = -2$, $(-2+2)^2 = 0$ & $(-2)^2 = 4$

$(x+2)^2 > x^2$ not true when $x = -2$.

(6) $(2n+1)^2 > 2^{n+1}$ for all pos integers < 7.

$n = 1$
$(2(1)+1)^2 > 2^{1+1}$
$9 > 4$

$n = 2$
$25 > 8$

$n = 3$
$49 > 16$

$n = 4$
$81 > 32$

$n = 5$
$121 > 64$

$n = 6$
$169 > 128$.

(7) $x^2 \geq x$

sometimes true - always true except for when $0 < x < 1$

(8) $(x+5)^2 - 4x - 9 = x^2 + 6x + 16$

$= (x+3)^2 - 9 + 16$

$= (x+3)^2 + 7 \geq 0$

so

$(x+5)^2 - 4x - 9 \geq 0 \rightarrow (x+5)^2 \geq 4x + 9$.

(9) Proof by exhaustion -

$n = 2 \rightarrow 2^2 + 2 = 6$
$n = 3 \rightarrow 3^2 + 2 = 11$
$n = 4 \rightarrow 4^2 + 2 = 18$
$n = 5 \rightarrow 5^2 + 2 = 27$
$n = 6 \rightarrow 6^2 + 2 = 38$
$n = 7 \rightarrow 7^2 + 2 = \cancel{5}\ 51$

none divisible by 4.