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6.5 → Circles and triangles

1a) AB: $m = \frac{\Delta y}{\Delta x}$

$m = \frac{4-6}{3--1} \rightarrow -\frac{1}{2} \rightarrow 2$

$y = mx + c$

midpoint of AB: $\left(\frac{-1+3}{2}, \frac{6+4}{2}\right)$

$= (1, 5)$

$5 = 2(1) + c$ $\quad \underline{y = 2x + 3}$

$5 - 2 = c$

$3 = c$

BC: $m = \frac{2-4}{5-3} \rightarrow -1 \rightarrow 1$

midpoint of BC: $\left(\frac{5+3}{2}, \frac{4+2}{2}\right)$

$= (4, 3)$

$3 = 1(4) + c$ $\quad \underline{y = x - 1}$

$3 = 4 + c$

$3 - 4 = c$

$-1 = c$

AC: $m = \frac{2-6}{5--1} \rightarrow -\frac{2}{3} \rightarrow \frac{3}{2}$

midpoint of AC: $\left(\frac{5+-1}{2}, \frac{2+6}{2}\right)$

$= (2, 4)$

$4 = \frac{3}{2}(2) + c$ $\quad \underline{y = \frac{3}{2}x + 1}$

$4 - 3 = c$

$1 = c$

1b) AB: $y = 2x + 3$

BC: $y = x - 1$

AC: $y = \frac{3}{2}x + 1$

$x - 1 = 2x + 3$

$-3 - 1 = 2x - x$

$-4 = x$

$y = 2(-4) + 3$

$y = -8 + 3$

$y = -5$

$(-4, -5)$

$y = \frac{3}{2}(-4) + 1$

$y = -5$

They all have a common point of $(-4, -5)$

1c) $(-4, -5)$ = centre of circle

$(x+4)^2 + (y+5)^2 = r^2$

$(5+4)^2 + (2+5)^2 = r^2$

$(9)^2 + (7)^2 = r^2$

$81 + 49 = 130$

equation: $(x+4)^2 + (y+5)^2 = 130$

2a) ~~PR~~ $= m_{PR} = \frac{15-1}{~~2~~ - -12} = 1$

$m_{RQ} = \frac{15-9}{2-8} = -1$

$m_{PR} \times m_{RQ} = -1$

$1 \times -1 = -1$

As the triangle PRQ is a right angle triangle, for PQ to be the diameter, PR and RQ should be perpendicular.

2b) ~~PR~~: $m_{PR} = 1 \rightarrow -1$

$m_{RQ} = -1 \rightarrow 1$

$y = mx + c$

midpoint of PR: $\left(\frac{-12+2}{2}, \frac{1+15}{2}\right)$

$\left(\frac{-10}{2}, \frac{16}{2}\right)$

$(-5, 8)$

$8 = -1(-5) + c$ $\quad y = -x + 3$ (PR)

$8 = 5 + c$

$8 - 5 = c$

$3 = c$

midpoint of RQ: $\left(\frac{2+8}{2}, \frac{15+9}{2}\right)$

$= (5, 12)$

$12 = 1(5) + c$ $\quad y = x + 7$ (RQ)

$12 - 5 = c$

$7 = c$

~~$y = x + 3$~~ $\quad x + 7 = -x + 3$

$2x = -4$

$x = -2$

$y = -2 + 7$

$y = 5$

$(-2, 5) \rightarrow$ centre

$(x+2)^2 + (y-5)^2 = r^2$

$(2+2)^2 + (15-5)^2 = r^2$

$4^2 + 10^2 = r^2$

$16 + 100 = 116$

equation: $(x+2)^2 + (y-5)^2 = 116$

3a) AB: $\sqrt{(1-3)^2 + (-2-6)^2}$

$\sqrt{(-2)^2 + (-8)^2}$

$\sqrt{4 + 64}$

$\sqrt{68}$

BC: $\sqrt{(-7-1)^2 + (0--2)^2}$

$\sqrt{(-8)^2 + (2)^2}$

$\sqrt{64 + 4}$

$\sqrt{68}$

AC: $\sqrt{(-7-3)^2 + (0-6)^2}$

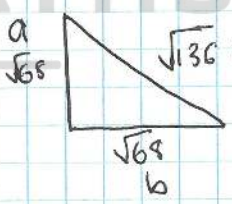
$\sqrt{(-10)^2 + (-6)^2}$

$\sqrt{100 + 36}$

$\sqrt{136}$

lengths AB and BC are the same which makes the triangle isosceles

$a^2 + b^2 = c^2$



$(\sqrt{68})^2 + (\sqrt{68})^2 =$

$68 + 68 = 136$

ABC is a right angle triangle as it fits the pythagoras theorem.

3b) ~~AB~~ $m_{AB} = \frac{1-3}{-2-6} = \frac{1}{4} \rightarrow -4$

$m_{BC} = \frac{-7-1}{0--2} = -4 \rightarrow \frac{1}{4}$

midpoint of AB: $\left(\frac{6+2}{2}, \frac{3+1}{2}\right)$

$(4, 2)$

midpoint of BC: $\left(\frac{-2+0}{2}, \frac{1+-7}{2}\right)$

$(-1, -3)$

3b) AC = diameter

m_{AC} ~~$\frac{7-3}{0-6}$~~ $\frac{4}{-6}$

midpoint of AC = centre

$\left(\frac{6+0}{2}, \frac{3+-7}{2}\right)$

$(3, -2)$

$(x-3)^2 + (y+2)^2 = r^2$

$(6-3)^2 + (3+2)^2 = r^2$

$(3)^2 + (5)^2 = r^2$

$9 + 25 = 34$

equation: $(x-3)^2 + (y+2)^2 = 34$

4a) ~~AB~~ $m_{AB} = \frac{10-14}{14-6} = -\frac{1}{2} \rightarrow 2$

midpoint of AB: $\left(\frac{6+14}{2}, \frac{14+10}{2}\right)$

$(10, 12)$

$12 = 2(10) + C$

$12 = 20 + C$

$12 - 20 = C$

$-8 = C$

$y = 2x - 8$

$y = -x + 10$

~~$-x = 2x$~~

$-x + 10 = 2x - 8$

$-2x - x = -8 - 10$

$-3x = -18$

$x = 6$

$y = 2(6) - 8$

$y = 12 - 8$

$y = 4$

$(6, 4) \rightarrow$ centre

$(x-6)^2 + (y-4)^2 = 100$

4b) $(x-6)^2 + (y-4)^2 = r^2$

$(6-6)^2 + (14-4)^2 = r$

$(0)^2 + (10)^2 = r^2$

$100 = r^2 \rightarrow$ equation = 6

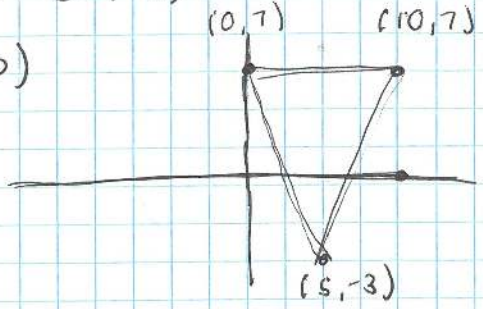
5a) $(x-5)^2 - 25 + (y+3)^2 - 9 = 91$

$(x-5)^2 + (y+3)^2 = 91 + 25 + 9$

$(x-5)^2 + (y+3)^2 = 125$

C = (5, -3)

5b)



Area = base × height × $\frac{1}{2}$

base = 10, height = 10

$\frac{10 \times 10}{2} = 50$

6a) $(x-2)^2 - 4 + (y+4)^2 - 16 = 33$

$(x-2)^2 + (y+4)^2 = 33 + 4 + 16$

$(x-2)^2 + (y+4)^2 = 53$

centre = (2, -4)

radius = $\sqrt{53}$

6b) $\sqrt{(-26 - -2)^2 + (9 - -5)^2}$

$\sqrt{(-4)^2 + (14)^2}$

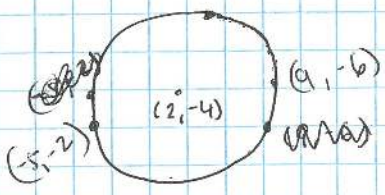
$\sqrt{16 + 196}$

$\sqrt{212}$

diameter = $2 \times \sqrt{53}$

diameter^2 = $4 \times 53 = 212$

6c)



Points PQR lie on the circle with RQ being the diameter

midpoint of PQ (centre) = (2, -4)

R = (0, r)

PR and PQR are perpendicular

$(0-2)^2 + (y+4)^2 = 53$

$(-2)^2 + (y+4)^2 = 53$

$4 + y^2 + 8y + 16 = 53$

$y^2 + 8y + 16 + 4 - 53 = 0$

$y^2 + 8y - 33 = 0$

$y = 3, y = -11$

↓

y is positive so 3

(0, 3)

7a) $(x+2)^2 - 4 + (y-3)^2 - 9 - 23 = 0$

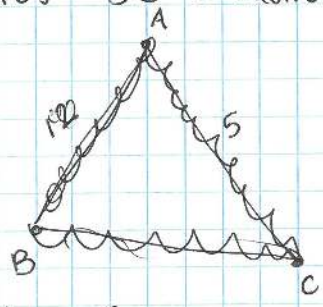
$(x+2)^2 + (y-3)^2 - 36 = 36$

centre: (-2, 3)

r = 6

7b) radius^2 = 36 ∴ radius = $\sqrt{36}$ = 6

7c)



As radius = 6
diameter (AB) = 12

So ABC is right angle triangle

$12^2 - 5^2 = BC^2$

$\sqrt{119} = BC$

10.9 (1dp)

8a) $m_{PQ} = \left(\frac{-1-3}{5--2}\right)$

$m_{PQ} = -\frac{4}{7}$

$m_{QR} = \frac{7}{4}$

$\frac{k--1}{-3-5} = \frac{7}{4}$

$\frac{k+1}{-8} = \frac{7}{4}$

$4(k+1) = 7 \times -8$

$4k + 4 = -56$

$4k = -56 - 4$

$4k = -60$

$k = -\frac{60}{4}$

$k = -15$

8b) midpoint of PQ:

$\left(\frac{-2+5}{2}, \frac{3+-1}{2}\right)$

$\left(\frac{3}{2}, 1\right)$

midpoint of QR:

$\left(\frac{5+-3}{2}, \frac{-1+-15}{2}\right)$

$(1, -8)$

PQ: $1 = \frac{7}{4}\left(\frac{3}{2}\right) + c$

$1 = \frac{21}{8} + c$

$1 - \frac{21}{8} = c$

$-\frac{13}{8}$

QR: $-8 = -\frac{4}{7}\left(\frac{5}{2}\right) + c$

$-8 = -\frac{6}{7} + c$

$-8 + \frac{6}{7} = c$

$-\frac{50}{7} = c$

$y = -\frac{4}{7}x - \frac{50}{7}$

$y = \frac{7}{4}x - \frac{13}{8}$

9a) midpoint of PR:

$\left(\frac{-4+k}{2}\right), \left(\frac{4+6}{2}\right)$

$\left(\frac{-4+k}{2}, 5\right)$

midpoint of PQ:

$\left(\frac{-4+8}{2}\right), \left(\frac{4+12}{2}\right)$

$\left(\frac{4}{2}, \frac{16}{2}\right)$

$(2, 8)$

$m_{PR} = \frac{12-4}{8--4} = \frac{2}{3}$

$\frac{6}{8-k} \times \frac{2}{3} = -1$

$\frac{12}{24-3k} = -1$

$12 = -24 + 3k$

$12 + 24 = 3k$

$36 = 3k$

$\frac{36}{3} = k$

$12 = k$

9b) $8 = -\frac{3}{2}(2) + c$

$8 + 3 = c$

$11 = c$

$PQ = y = -\frac{3}{2}x + 11$

QR midpoint

$\left(\frac{8+12}{2}, \frac{12+6}{2}\right)$

$(10, 9)$

$9 = \frac{2}{3}(10) + c$

$9 = \frac{20}{3} + c$

$9 - \frac{20}{3} = c$

$\frac{7}{3} = c$

$y = \frac{2}{3}x + \frac{7}{3}$

$x = \frac{52}{5}$ $y = -\frac{23}{5}$