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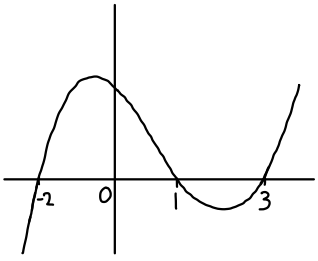
Problem Solving: Set A

Bronze



$y = x^3 + bx^2 + cx + d$

a) Graph crosses -2, 1 and 3 on x-axis

$\therefore (x+2), (x-1)$ and $(x-3)$

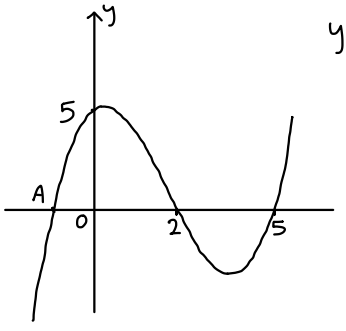
Expand: $x^3 - 2x^2 - 5x + 6$

$\therefore b = -2,\ c = -5,\ d = 6$

b) Crosses 6 at y-axis

$\therefore$ Crosses at $(0, 6)$

Silver



$y = x^3 + bx^2 + cx + d$

$x = -A, \quad x = 2, \quad x = 5$

$(x + A) \quad (x - 2) \quad (x - 5)$

y-intercept $= (0, 5) \Rightarrow x = 0$

$d = 5$

$$\begin{aligned} x^3 + bx^2 + cx + 5 &= (x-2)(x-5)(x+A) \\ &= (x^2 - 7x + 10)(x + A) \\ &= x^3 + Ax^2 - 7x^2 - 7Ax + 10x + 10A \end{aligned}$$

$$x^3 + bx^2 + cx + 5 \equiv x^3 + (A-7)x^2 + (10 - 7A)x + 10A$$

Equate coefficients :

$10A = 5$ $\qquad$ $10 - 7A = C$

$A = \frac{1}{2}$ $\qquad$ $10 - 7(\frac{1}{2}) = C$

$\frac{13}{2} = C$

$A - 7 = b$

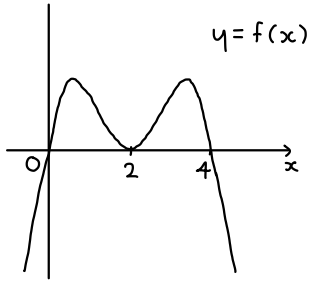
$\frac{1}{2} - 7 = b$

$-\frac{13}{2} = b$

$\therefore \quad y = x^3 - \frac{13}{2}x^2 + \frac{13}{2} + 5$

where $b = -\frac{13}{2}$, $c = \frac{13}{2}$ and $d = 5$

Gold



$f(x) = ax^4 + bx^3 + cx^2 + dx$

a) $y = a(x^4 + mx^3 + nx^2 + px)$

$x = 2 \Rightarrow$ double root

$\therefore a(x)(x-2)(x-2)(x-4)$

$= a(x)(x^2 - 4x + 4)(x-4)$

$= a(x)(x^3 - 8x^2 + 20x - 16)$

$a(x^4 - 8x^3 + 20x^2 - 16x)$

$m = -8, \quad n = 20, \quad p = -16$

b) $f(x) = 8$ has solutions $x = 2 \pm \sqrt{2}$

Sub $x = 2 \pm \sqrt{2}$ into $x^4 - 8x^3 + 20x^2 - 16x$

$(2 - \sqrt{2})^4 - 8(2 - \sqrt{2})^3 + 20(2 - \sqrt{2})^2 - 16(2 - \sqrt{2}) = -4$

$(2 + \sqrt{2})^4 - 8(2 + \sqrt{2})^3 + 20(2 + \sqrt{2})^2 - 16(2 + \sqrt{2}) = -4$

Both $2 + \sqrt{2}$ and $2 - \sqrt{2}$ yield the same result

Solve for a:

$a(-4) = 8$

$a = \frac{8}{-4}$

$a = -2$

$\therefore -2(x^4 - 8x^3 + 20x^2 - 16x)$

$-2x^4 + 16x^3 - 40x^2 + 32x$