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3.5 Quadratic Inequalities

1.

a) $x^2 < 25$

$(x-5)(x+5) < 0$

$-5 < x < 5$

b) $x(2x+1) < 3$

$x(2x+1) - 3 < 0$

$2x^2 + x - 3 < 0$

$(2x+3)(x-1) < 0$

$-\frac{3}{2} < x < 1$

c) $0 > 1 - 3x^2 - 2x$

$0 > 3x^2 - 2x - 1$

$0 > (3x-1)(x+1)$

$x < -1, \quad x > \frac{1}{3}$

d) $2x^2 \geq 5x$

$2x^2 - 5x \geq 0$

$x(2x-5) \geq 0$

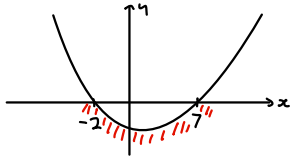
$x \leq 0, \quad x \geq \frac{5}{2}$

2.

a) $x^2 - 5x - 14 < 0$ and $5x + 2 > 12$

$(x-7)(x+2)$

$x = 7 \quad x = -2$

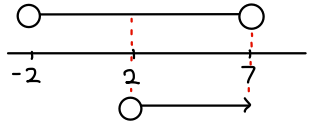


$-2 < x < 7$

$5x + 2 > 12$

$5x > 10$

$x > 2$

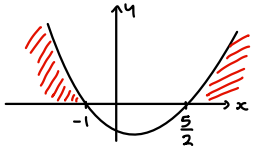


$\{x : 2 < x < 7\}$

2 b) $2x^2 - 3x - 5 > 0$ and $x^2 + 3x - 4 < 0$

$(2x-5)(x+1)$

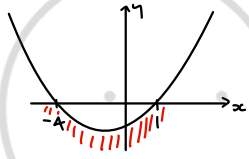
$x = \frac{5}{2}$ $x = -1$



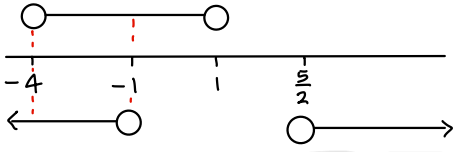
$x < -1$ or $x > \frac{5}{2}$

$(x-1)(x+4)$

$x = 1$ $x = -4$



$-4 < x < 1$

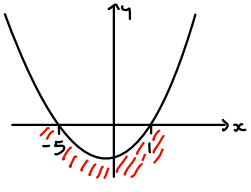


$\{x : -4 < x < -1\}$

c) $3x^2 + 12x - 15 < 0$ and $x^2 + 5x + 6 > 0$

$(x-1)(x+5)$

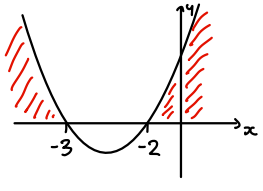
$x = 1$ $x = -5$



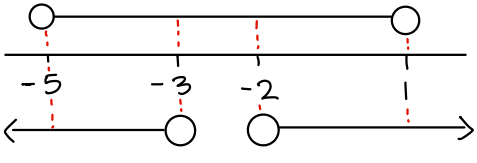
$-5 < x < 1$

$(x+2)(x+3)$

$x = -2$ $x = -3$



$x < -3$ or $x > -2$



$\{x : -5 < x - 3\} \cup \{x : -2 < x < 1\}$

3.

a) $36 > \frac{1}{x^2}$

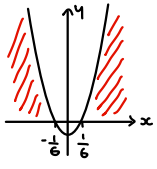
Multiply both sides by x^2

$36x^2 > 1$

$36x^2 - 1 > 0$

$(6x-1)(6x+1) = 0$

$x = \frac{1}{6}$ or $x = -\frac{1}{6}$



$x < -\frac{1}{6}$, $x > \frac{1}{6}$

b) $3 < \frac{5}{x}$

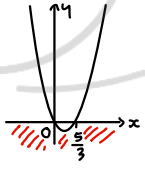
Multiply both sides by x^2

$3x^2 < 5x$

$3x^2 - 5x < 0$

$x(3x-5) = 0$

$x = 0$ or $x = \frac{5}{3}$



$0 < x < \frac{5}{3}$

c) $16 + \frac{6}{x} \geq \frac{2}{x}$

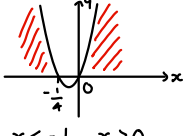
$16 + \frac{4}{x} \geq 0$

Multiply both sides by x^2

$16x^2 + 4x \geqslant 0$

$4x(4x+1) = 0$

$x = 0$ or $x = -\frac{1}{4}$



$x \leq -\frac{1}{4}$, $x > 0$

4. $x^2-6x+11 = (x-3)^2+2 \implies x^2-6x+11 \geqslant 2 > 0$

5. a) $2x-2>7-x$

$3x>9$

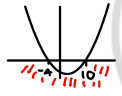
$x>3$

b) $x^2-6x \leqslant 40$

$x^2-6x-40 \leqslant 0$

$(x-10)(x+4)=0$

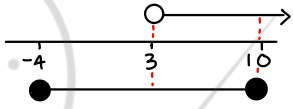
$x=10$ or $x=-4$



$-4 \leqslant x \leqslant 10$

c) $2x-2>7-x$ and $x^2-6x \leqslant 40$

$x>3$ $\quad -4 \leqslant x \leqslant 10$



$3 < x \leqslant 10$

6. a) $x^2+(k-2)x+(4-2k)=0$

$a=1$

$b=k-2$ Use b^2-4ac

$c=4-2k$

$(k-2)^2-4\times1\times(4-2k)>0$

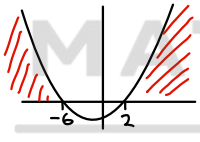
$k^2-4k+4-16+8k>0$

therefore $k^2+4k-12>0$

b) $k^2+4k-12>0$

$(k-2)(k+6)=0$

$k=2$ $\quad k=-6$



$k<-6,\ k>2$

7. a) $2(5x+2)+2(6x)$

$10x+4+12x$

$22x+4>103$

$22x>99$

$x>4.5$

b) $3x(5x+2)+3x(2x+1)$

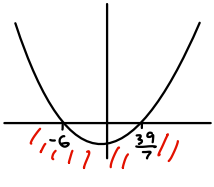
$15x^2+6x+6x^2+3x$

$21x^2+9x<702$

$21x^2+9x-702<0$

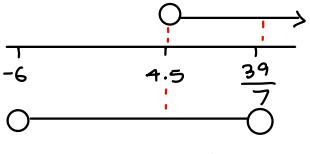
$(x-\frac{39}{7})(x+6)=0$

$x=\frac{39}{7}$ or $x=-6$



$-6<x<\frac{39}{7}$

c)



$4.5<x<\frac{39}{7}$

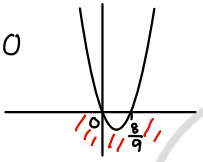
8. $2x^2 - 3kx + k = 0$

No real roots so $b^2 - 4ac < 0$

$a = 2$
$b = -3k$
$c = k$

$(-3k)^2 - 4 \times 2 \times k$

$9k^2 - 8k = 0$
$k(9k - 8) = 0$
$k = 0$ or $k = \frac{8}{9}$



$0 < k < \frac{8}{9}$

9. Curve C, $y = x^2 + 3px + 8p$

line L, $y = 1 - 2x$

$x^2 + 3px + 8p = 1 - 2x$
$x^2 + 3px + 2x + 8p - 1 = 0$
$x^2 + (3p + 2)x + (8p - 1) = 0$

Line doesn't intersect curve so $b^2 - 4ac < 0$

$a = 1$
$b = 3p + 2$
$c = 8p - 1$

$(3p + 2)^2 - 4 \times 1 \times (8p - 1)$

$9p^2 + 12p + 4 - 32p + 4$

$9p^2 - 20p + 8$

$p = \frac{10 + 2\sqrt{7}}{9}$ or $p = \frac{10 - 2\sqrt{7}}{9}$

$9p^2 - 20p + 8$ opens upwards so it is less than zero between the roots:

$\frac{10 - 2\sqrt{7}}{9} < p < \frac{10 - 2\sqrt{7}}{9}$

10. $\frac{3}{x-1} < 2, \quad x \neq 1$

Multiply both sides by $(x-1)^2$

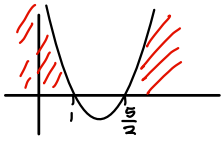
$3(x-1) < 2(x-1)^2$

$3x - 3 < 2x^2 - 4x + 2$

$2x^2 - 7x + 5 > 0$

$(2x-5)(x-1) = 0$

$x = \frac{5}{2}$ or $x = 1$



$x < 1$ or $x > 2.5$